University of Anbar

College of Engineering

Civil Engineering Department

LECTURE NOTE
COURSE CODE- CE 2308
ENGINEERING SURVEYING

Chapter One

By

Dr. Maher Shakir Mahmood **Dr. Hameed Aswed**

Engineering Surveying I

This course introduces knowledge about chain surveying, compass surveying, plane table distance measurements, levelling and traversing.

References:

- Charles D. Ghilani, Paul R. Wolf, Elementary Surveying, Prentice Hall, 12th ed., 2008.
- Chandra, A. M. Surveying Problem Solution with Theory and Objective Type Questions. New Age International, 2005.
- ياسين عبيد ،المساحة الهندسية.

COURSE TOPICS:

Chapter One: Basic Principle of Surveying

Chapter Two: Distance Measurements Using Tape

Chapter Three: Leveling—Theory and Methods

Chapter Four: Distance Measurements Using Trigonometric & EDM

Chapter Five: Angles, Azimuth, and Bearing

Chapter Six: Traversing

PROGRAM AND COURSE OUTCOMES:

1. Students will obtain knowledge in:
    a. Mathematics: Students in this class will use basic mathematical skills in real world calculations.
    b. Science: Scientific procedures in surveying show the student the necessity of redundant information and methods for determining and evaluating errors.
    c. Engineering: Surveying is one of the original and most recognized civil engineering skills.
2. Expose students to state-of-the-art and state-of-the-practice facilities and equipment.
3. Students will learn to use equipment similar in type and quality to those professional surveyors use in their businesses.

CHAPTER 1
BASIC PRINCIPLE OF SURVEYING

Definition of Surveying:

- Surveying is defined as "taking a general view, by observation and measurement determining the boundaries, size, position, quantity, condition, value etc. of land, estates, building, farms mines etc. and finally presenting the survey data in a suitable form". This covers the work of the valuation surveyor, the quantity surveyor, the building surveyor, the mining surveyor and so forth, as well as the land surveyor.
- The art of making measurements of the relative positions of natural and man-made features on the Earth's surface, and the presentation of this information either graphically or numerically.

Types of Surveys:

1. Geodetic Surveying: The type of surveying that takes into account the true shape of the earth. These surveys are of high precision and extend over large areas.
2. Plane Surveying: The type of surveying in which the mean surface of the earth is considered as a plane, or in which its spheroidal shape is neglected, with regard to horizontal distances and directions.

History of Surveying

- The oldest historical records in the subject of surveying began in Egypt. Egyptians divided the land of Egypt into plots for the purpose of taxation and boundaries of annual floods.
- The first surveying works date back to the antiquity, the Greek provided the first account of surveying techniques.

Specialized Types of Surveys:

Many types of surveys are so specialized that a person proficient in a particular discipline may have little contact with the other areas. Some important classifications are described briefly here.

- **Control surveys** establish a network of horizontal and vertical monuments that serve as a reference framework for initiating other surveys.
- **Topographic surveys** determine locations of natural and artificial features and elevations used in map making.
- **Land, boundary, and cadastral surveys** establish property lines and property corner markers. The term cadastral is now generally applied to surveys of the public lands systems.
- **Hydrographic surveys** define shorelines and depths of lakes, streams, oceans, reservoirs, and other bodies of water.
- **Alignment surveys** are made to plan, design, and construct highways, railroads, pipelines, and other linear projects.
- **Construction surveys** provide line, grade, control elevations, horizontal positions, dimensions, and configurations for construction operations. They also secure essential data for computing construction pay quantities.
- **As-built surveys** document the precise final locations and layouts of engineering works and record any design changes that may have been incorporated into the construction.
- **Mine surveys** are performed above and below ground to guide tunneling and other operations associated with mining. This classification also includes geophysical surveys for mineral and energy resource exploration.
- **Solar surveys** map property boundaries, solar easements, obstructions according to sun angles, and meet other requirements of zoning boards and title insurance companies.
- **Optical tooling (also referred to as industrial surveying or optical alignment)** is a method of making extremely accurate measurements for manufacturing processes where small tolerances are required.

Except for control surveys, most other types described are usually performed using plane-surveying procedures, but geodetic methods may be employed on the others if a survey covers an extensive area or requires extreme accuracy.

Ground, aerial, and satellite surveys are broad classifications sometimes used.

A. Ground surveys utilize measurements made with ground-based equipment such as automatic levels and total station instruments.
B. Aerial surveys are accomplished using either photogrammetry or remote sensing. Photogrammetry uses cameras that are carried usually in airplanes to obtain images, whereas remote sensing employs cameras and other types of sensors that can be transported in either aircraft or satellites.
C. Satellite surveys include the determination of ground locations from measurements made to satellites using global navigation satellite systems (GNSS) receivers, or the use of satellite images for mapping and monitoring large regions of the Earth.

The Role of Surveyors in Civil Engineering Practice

- to maintain the geometric order during the construction process.
- to provide fundamental data for the design and planning process.
- to provide quantity control during the construction process (for example: earthwork quantities).
- to monitor the structure after the construction (deflection, deformations, etc.).

Basic Measurements and Instruments

Surveying consists of four basic measurements:

1. Horizontal distance – measured by tapes, chains, tacheometers, EDM, pacing, odometer, etc.
2. Vertical distance – measured by levels, tacheometers.
3. Horizontal angles – measured by theodolites, compasses.
4. Vertical angles – measured by theodolites, clinometers.

Units of Measurement

There are two principal measurements in surveying works:

1. Linear measurements: the basic unit used is the meter. Decimal fractions of the meter are also used when accuracy is required. Other units include the foot, the inch, the yard, the mile, etc.

2. Angular measurements: even though the basic unit is the radian, the degree is used in surveying field works. For accuracy subdivisions of the degree are available, viz. the minute and the second. 1 degree (1°) = 60 minutes

   1 minute (1') = 60 seconds (")

   360 degrees = 400 gons

   1 gon = 100 centecimal unit

   2π rad = 360 degrees

- Conversions between the British/American and metric systems

  1 inch = 25.4 mm 1 foot = 0.3048 m 1 yard = 0.9144 m

  1 mile = 1.609344 Km 1 acre = 4046.8564224 m^2

Scales

The scale of a map or a plan is the ratio of a distance measured on the plan or map to its corresponding distance on the ground. Example 1:100, 1:10,000. Scale primarily depends on the type of the work done (the accuracy with which a distance is to be transferred from the map or the plan). In general, scales may be categorized as follows: For maps

a. Large scales < 1:200
b. Intermediate scales 1:2000 to 1:10,000
c. Small scales 1:10,000 to 1:100,000,000 For plans
d. Site plans 1:50 to 1:500
e. Detail plans 1:1 to 1:20

A scale bar or a graphical scale is another form of indicating the scale of a drawing. It usually appears with numerical scales on the drawing sheet.

Plan and Map

The difference between a map and a plan is that on a map the scale is too small to allow every feature to be properly represented to scale. Thus, conventional symbols are used to represent features that would otherwise be too small to be recognized. A plan, on the other hand, shows all features on the ground correctly to scale.

Theory of Errors

Measurement is an observation carried out to determine the values of quantities (distances, angles, directions, temperature …) the process of taking measurements involves physical operations like setting up, calibrating, pointing matching comparing etc of the instrument. The fundamental principle of measurement of surveying is that no measurement is exact and the true value of quantity being measured is never known. No matter how a sophisticated instrument used and all necessary cares are taken the result of a survey measurement will contain some error. This is due to:

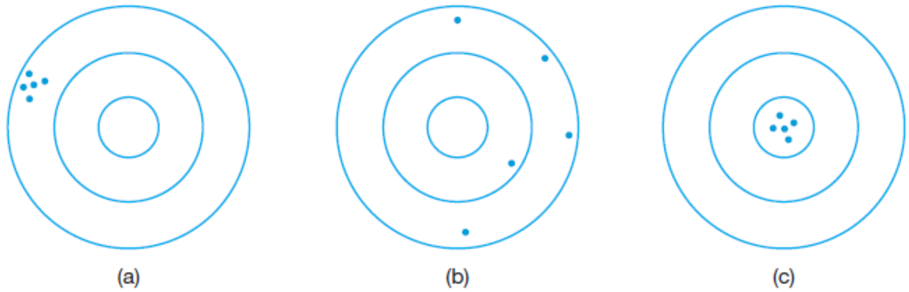
- Lack of perfection by the surveyor in his senses of seeing touching, hearing.
- Imperfection by the instruments and methods (in construction and adjustments and environmental factors in their operation and approximations etc).

Therefore, it is theoretically impossible to get the ”exact” value or ”true” value of any measured quantity as all of our measurements will contain some error and as the “ exact” value should contain infinite significant digits (Which is practically impossible). Hence one may ask why so measuring, as we cannot get the “exact “value of a measured quantity? In surveying and generally in any scientific measurements what is important is not the “exact’’ value rather the possibility of carrying out the measurement to the degree of accuracy sufficient to the desired purpose. Hence, as long as the desired accuracy is achieved our measurements can be used as an equivalent to the “true “value.

Measures of quality

- **Accuracy**: is a parameter indicating the closeness of measured value to the “true’’ or ‘’exact” value of a quantity. It indicates the degree of perfection obtained in measurements. The further a measured value from the its “true” value the less accurate it is.
- **Precision**: is the degree of refinement with which a given quantity is measured. In other words, it is the closeness of the measured values to one another regardless of their closeness to the true value.

Examples of precision and accuracy. (a) Results are precise but not accurate. (b) Results are neither precise nor accurate. (c) Results are both precise and accurate.



It is possible for surveyors to obtain both accuracy and precision by exercising care, and using good instruments and procedures.

In measuring distance, precision is defined as the ration of the error of the measurement to the distance measured. Note a measurement can be accurate but not precise; precise but not accurate; precise and accurate; and neither precise nor accurate as shown in figure.

Error

Error is generally defined, as the deviation of the measured value from the "exact" value of a quantity. The study of errors is important in surveying as it helps the surveyor understand the sources and exercise the necessary care and apply correction to minimize their effect so that an acceptable accuracy is achieved.

Error Sources

Generally the errors in surveying measurements are classified as:

a. **Personal**: the error that occurs due to lack of perfection in the surveyor's sense of sight, touch, hearing etc. during survey activity. Also mistakes due to carelessness or fatigue of the surveyor are classed under this category. This type of error can be minimized with care and vigilance by the part of the surveyor.

b. **Instrumental**: it is the error type that occurs due to imperfection of the instruments in manufacture and during adjustments and due to wear and tear by usage. Also included are mistakes due to failure or damage of the instrument. This type of error can be minimized with careful handling, maintenance and adjustment and calibration of instruments and by applying corrections.

c. **Natural**: included under this are errors due to effect of temp, pressure, humidity, magnetic variation etc. This type of error can be minimized by applying correction and by carrying out the survey when their effect is minimal.

Error Types

Classically errors are classified in to three; these are Mistakes, Systematic errors and Random errors.

1. **Mistakes**: These actually are not error because they usually are so gross in magnitude compared to the other two types. These are rather blunder made by surveyor or his equipment and can occur at any stage of the survey (during reading, recording computing and plotting).

Source: May be due to one of the following:

- Carelessness or fatigue by surveyor
- Failure of equipment

Examples are:

Reading wrong scale

- ➢ Transposing figure in recording ex 56 instead of 65 or reading 6 instead of 9.
- ➢ Omitting digits during recording ex 200 instead of 2000

Sighting towards wrong target etc.

2. **Systematic errors**: These are error types with relatively small magnitude compared to mistakes, and are result of some systems whose effect can be expressed in mathematical relations; hence their magnitude and sign can be estimated (determined). In most cases, the system causing the systematic error can be personal, instrumental or physical and environmental conditions or may be result of choice of geometric or mathematical model used. Systematic / cumulative errors are those which for constant conditions remain the same as to sign and magnitude: hence mare repetition will not help in detection and elimination. As their values can be determined correction can be applied to improve the data. In addition, proper calibration and adjustment of instruments also contribute to minimizing their effect.

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LECTURE NOTE
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Chapter Two

By

Dr. Maher Shakir **Dr. Hameed Aswad**

CHAPTER TWO

DISTANCE MEASUREMENTS USING TAPE

CHAIN SURVEYING

This is the simplest and oldest form of land surveying of an area using linear measurements only. It can be defined as the process of taking direct measurement, although not necessarily with a chain.

Equipment Used in Chain Surveying

These equipment can be divided into three, namely

a) Those used for linear measurement. (Chain, steel band, linear tape)

b) Those used for slope angle measurement and for measuring right angle (Eg. Abney level, clinometer, cross staff, optical squares)

c) Other items (Ranging rods or poles, arrows, pegs etc).

1. **Chain:-**

   The chain is usually made of steel wire, and consists of long links joined by shorter links. It is designed for hard usage, and is sufficiently accurate for measuring the chain lines and offsets of small surveys.

   - They are made up of links which measure 200mm from centre to centre of each middle connecting ring and surveying brass handless are fitted at each end.

2. **Steel Bands:**

   This may be 30m, 50m or 100m long and 13mm wide. It has handles similar to those on the chain and is wound on a steel cross. It is more accurate but less robust than the chain. The operating tension and temperature for which it was graduated should be indicated on the band.

3. **Tapes:**

   Tapes are used where greater accuracy of measurements are required, such as the setting out of buildings and roads. They are 15m or 30m long marked in metres, centimeter and millimeters. Tapes are classified into three types;

i. Linen or Linen with steel wire woven into the fabric;

ii. Fibre Glass Tapes

iii. Steel tapes:

4. **Arrows:**

They comprise of a piece of steel wire about 0.5m long, and are used for marking temporary stations.

5. **Pegs**

They are made of wood 50mm x 50mm and some convenient length. They are used for points which are necessary to be permanently marked, like intersection points of survey lines.

6. **Ranging Rod:**

These are poles of circular section 2m, 2.5m or 3m long, painted with characteristic red and white bands which are usually 0.5m long and tipped with a pointed steel shoe to enable them to be driven into the ground. They are used in the measurement of lines with the tape, and for marking any points which need to be seen.

7. **Optical Square:**

   This instrument is used for setting out lines at right angle to main chain line. It is used where greater accuracy is required. There are two types of optical square, one using two mirrors and the other a prism.

GENERAL PROCEDURE IN MAKING A CHAIN SURVEY

a) **Reconnaissance**: Walk over the area to be surveyed and note the general layout, the position of features and the shape of the area.
b) **Choice of Stations:** Decide upon the framework to be used and drive in the station pegs to mark the stations selected.
c) **Station Marking*:*** Station marks, where possible should be tied - in to a permanent objects so that they may be easily replaced if moved or easily found during the survey. In soft ground wooden pegs may be used while rails may be used on roads or hard surfaces.
d) **Witnessing**: This consists of making a sketch of the immediate area around the station showing existing permanent features, the position of the stations and its description and designation. Measurements are then made from at least three surrounding features to the station point and recorded on the sketch. The aim of witnessing is to re-locate a station again at much later date even by others after a long interval.
e) **Offsetting*:-*** Offsets are usually taken perpendicular to chain lines in order to dodge obstacles on the chain line.

f) **Sketching:-** the layout on the last page of the chain book, together with the date and the name of the surveyor, the longest line of the survey is usually taken as the base line and is measured first.

Corrections to Linear Measurement and their Application:-

The corrections are to be applied to the linear measurements with a chain or a tape where such accuracy is required. Systematic errors in taping are due to:
(1) incorrect tape length, (2) tape not horizontal, (3) fluctuation in temperature of the tape, (4) in correct tension or pull, (5) sag in tape, (6) incorrect alignment and (7) tape not straight.

1. Correction for Absolute Length

Due to manufacturing defects the absolute length of the tape may be different from its designated or nominal length. Also with use the tape may stretch causing change in the length and it is zaz/ that the tape is regularly checked under standard conditions to determine its absolute length. The correction for absolute length or standardization is given by

$$C_a = \frac{c}{l} L$$

where
c = the correction per tape length,
l = the designated or nominal length of the tape, and
L= the measured length of the line.
If the absolute length is more than the nominal length the sign of the correction is positive and vice
versa.

2. Correction for Temperature

If the tape is used at a field temperature different from the standardization temperature then the temperature correction to the measured length is

$$c_t = \alpha(t_m - t_0)L$$

where
α = the coefficient of thermal expansion of the tape material,

t_m = the mean field temperature, and

t_0 = the standardization temperature.

The sign of the correction takes the sign of $(t_m - t_0)$.

3. Correction for Pull or Tension

If the pull applied to the tape in the field is different from the standardization pull, the pull correction is to be applied to the measured length. This correction is

$$c_p = \frac{P - P_0}{AE} L$$

where

P = the pull applied during the measurement,

P_0 = the standardization pull,

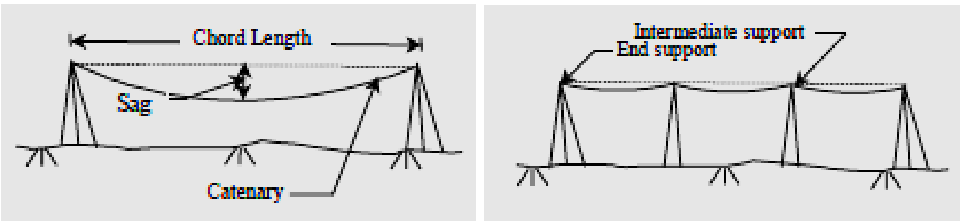
A = the area of cross-section of the tape, and

E = the Young's modulus for the tape material.

The sign of the correction is same as that of $(P - P_0)$.

4. Correction for Sag

For very accurate measurements, the tape can be allowed to hang in catenary between two supports. In the case of long tape, intermediate supports as shown in the below figure, can be used to reduce the magnitude of correction.



The tape hanging between two supports, free of ground, sags under its own weight, with maximum dip occurring at the middle of the tape. This necessitates a correction for sag if the tape has been standardized on the flat, to reduce the curved length to the chord length. The correction for the sag is

$$c_g = \frac{1}{24}\left[\frac{W}{P}\right]^2 L$$

where

W = the weight of the tape per span length.

The sign of this correction is always negative.

If both the ends of the tape are not at the same level, a further correction due to slope is required. It is given by

$$c'_g = c_g \cos \alpha$$

where

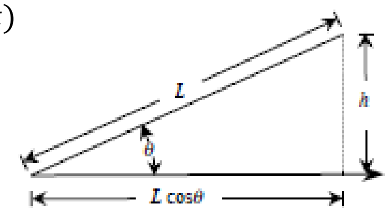
α = the angle of slope between the end supports.

5. **Correction for Slope**

If the length L is measured on the slope as shown in the figure, it must be reduced to its horizontal equivalent L cos θ. The required slope correction is

$$c_s = (1 - \cos \theta)\, L \qquad (exact)$$

$$c_s = \frac{h^2}{2L} \qquad (approximate)$$



where

θ = the angle of the slope, and

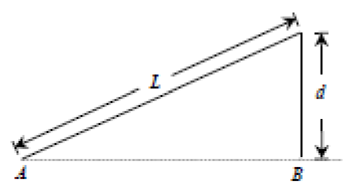
h = the difference in elevation of the ends of the tape.

The sign of this correction is always negative.

6. **Correction for Alignment**

If the intermediate points are not in correct alignment with ends of the line, a correction for alignment given below, is applied to the measured length.

$$c_m = \frac{d^2}{2L} \qquad (approximate)$$



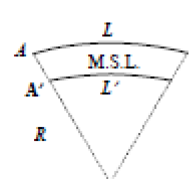
where

d = the distance by which the other end of the tape is out of alignment.

The correction for alignment is always negative.

7. **Reduction to Mean Sea Level (M.S.L.)**

In the case of long lines in triangulation surveys, the relationship between the length AB measured on the ground and the equivalent length A′B′ at mean sea level has to be considered. Determination of the equivalent mean sea level length of the measured length is known as reduction to mean sea level. The reduced length at mean sea level is given by



$$\acute{L} = \frac{R}{(R+H)}$$

where $c_{msi} = \frac{HL}{R}$ (*approximate*)

R = the mean earth's radius (6372 km), and

H = the average elevation of the line.

When H is considered small compared to R, the correction to L is given as

$$c_{msi} = \frac{HL}{R} \qquad (approximate)$$

The sign of the correction is always negative.

The various tape corrections discussed above, are summarized in the below.

Correction	Sign	Formula
Absolute length (c_a)	±	$\frac{c}{l}L$
Temperature (c_t)	±	$\alpha(t_m - t_0)L$
Pull (c_p)	±	$\frac{(P-P_0)}{AE}L$
Sag (c_g)	–	$\frac{1}{24}\left(\frac{W}{P}\right)^2 L$
Slope (c_s)	–	$(1-\cos\theta)L$ (*exact*)
Alignment (c_m)	–	$\frac{h^2}{2L}$ (*approximate*)
Mean sea level (c_{msl})	–	$\frac{d^2}{2L}$ (*approximate*)
		$\frac{HL}{R}$ (*approximate*)

Example 2.1: A line AB between the stations *A* and *B* was measured as 348.28 using a 20 m tape, too short by 0.05 m. Determine the correct length of *AB*, the reduced horizontal length of

AB if *AB* lay on a slope of 1 in 25, and the reading required to produce a horizontal distance of 22.86 m between two pegs, one being 0.56 m above the other.

Solution:

(a) Since the tape is too short by 0.05 m, actual length of AB will be less than the measure length. The correction required to the measured length is

$$C_a = \frac{c}{l} L$$

It is given that

c = 0.05 m, l = 20 m, L=348.28 m

$$C_a = \frac{0.05}{20} \times 348.28 = 0.87\ m$$

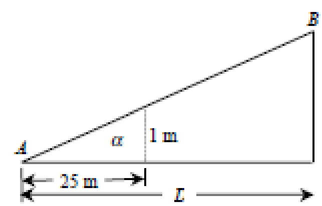
The correct length of the line

$$= 348.28 - 0.87 = 347.41\ m$$

(b) A slope of 1 in 25 implies that there is a rise of 1 m for every 25 m horizontal distance. If the angle of slope is α then

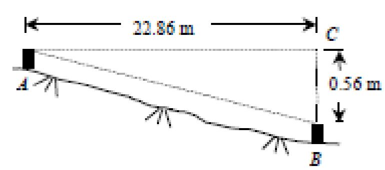
$$\tan\alpha = \frac{1}{25}$$

$$\alpha = tan^{-1}\frac{1}{25} = 2^\circ 17' 26''$$



Thus the horizontal equivalent of the corrected slope length 347.41 m is

$$D = AB\cos\alpha = 347.41 \times \cos\left(2^\circ 17' 26''\right) = 347.13\ m$$



We have

$$AB = \sqrt{Ac^2 + CB^2} = \sqrt{(22.86)^2 + (0.56)^2} = 22.87$$

Therefore the reading required

$$= 22.87 + \frac{0.05}{20.0} \times 22.87 = \mathbf{22.93\ m}$$

Example 2.2: A tape of standard length 20 m at 85°F was used to measure a base line. The measured distance was 882.50 m. The following being the slopes for the various segments of the line:

Segment length (m)	Slope
100	2°20′
150	4°12′
50	1°06′
200	7°48′
300	3°00′
82.5	5°10′

Calculate the true length of the line if the mean temperature during measurement was 63°F and the coefficient of thermal expansion of the tape material is 6.5×10^{-6} per °F.

Solution:

Correction for temperature

$$c_t = \alpha (t_m - t_0) L$$

$$= 6.5 \times 10^{-6} \times (63 - 85) \times 882.50 = -0.126 \text{ m}$$

Correction for slope

$$c_s = \Sigma[(1 - \cos\alpha) L]$$

$$= (1 - \cos 2^\circ 20') \times 100 + (1 - \cos 4^\circ 12') \times 150 + (1 - \cos 1^\circ 06') \times 50 + (1 - \cos 7^\circ 48') \times 200 + (1 - \cos 3^\circ 00') \times 300 + (1 - \cos 5^\circ 10') \times 82.5$$

$$= -3.092 \text{ m}$$

$$\text{Total correction} = c_t + c_s = -0.126 + (-3.092) = -3.218 \text{ m}$$

$$\text{Correct length} = 882.50 - 3.218 = \mathbf{879.282 \text{ m}}$$

Example 2.3: A base line was measured by tape suspended in catenary under a pull of 145 N, the mean temperature being 14°C. The lengths of various segments of the tape and the difference in level of the two ends of a segment are given in the below table.

Bay/Span	Length (m)	Difference in level (m)
1	29.988	+ 0.346
2	29.895	– 0.214
3	29.838	+ 0.309
4	29.910	– 0.106

If the tape was standardized on the flat under a pull of 95 N at 18°C determine the correct length of the line. Take Cross-sectional area of the tape = 3.35 mm2; Mass of the tape = 0.025 kg/m; Coefficient of linear expansion = 0.9×10^{-6} per °C; Young's modulus = 14.8×10^{4} MN/m^2; Mean height of the line above M.S.L. = 51.76 m; Radius of earth = 6370 km.

Solution:

P0 = 95 N, P = 145 N

t0 = 18°C, tm = 14°C

A = 3.35 mm2, α = 0.9 × 10–6 per °C

w = mg = 0.025 × 9.81 kg/m

E = 14.8 × 104 MN/m2 = $\frac{14.8\times10^4\times10^6}{10^6}$N/mm2 = 14.8 × 104 N/mm2

H = 51.76 m, R = 6370 km

Total length of the tape L = 29.988 + 29.895 + 29.838 + 29.910 = 119.631 m

Temperature correction

$c_t = \alpha(t_m - t_0)L$ = 0.9 × 10–6 × (14 – 18) × 119.631 = – 0.0004 m

Pull correction

$$c_p = \frac{(P - P_0)L}{AE} = \frac{(145\ -\ 95)\times\ 119.631}{3.35\ \times 14.8\ \times 10^4} = 0.0121\text{ m}$$

Sag correction

$$c_g = -\frac{1}{24}\left(\frac{W}{P}\right)^2 L = -\left[\frac{1}{24}\left(\frac{wl_1}{P}\right)^2 l_1 + \frac{1}{24}\left(\frac{wl_2}{P}\right)^2 l_2 + \frac{1}{24}\left(\frac{wl_3}{P}\right)^2 l_3 + \frac{1}{24}\left(\frac{wl_4}{P}\right)^2 l_4\right]$$

$$= -\frac{w^2}{24\ P^2}\left(l_1{}^3 + l_2{}^3 + l_3{}^3 + l_4{}^3\right)$$

$$= -\frac{(0.025 \times 9.81)^2}{24\ 145^2}(29.988^3 + 29.895^3 + 29.838^3 + 29.910^3) = -\ 0.0128\ \text{m}$$

Slope correction

$$c_s = -\frac{h^2}{2L} = -\frac{1}{2}\left(\frac{0.346^2}{29.988} + \frac{0.214^2}{29.895} + \frac{0.309^2}{29.838} + \frac{0.106^2}{29.910}\right) = -\ 0.0045\ \text{m}$$

M.S.L. correction

$$c_{msl} = -\frac{HL}{R} = -\frac{51.76\ \times 119.631}{6370\ \times 1000} = -\ 0.0010\ \text{m}$$

Total correction = ct + cp + cg + cs + cmsl

= – 0.0004 + 0.0121 – 0.0128 – 0.0045 – 0.0010 = – 0.0066 m

Correct length = 119.631 – 0.0066 = 119.624 m

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LECTURE NOTE
ENGINEERING SURVEYING

Chapter Three

By

Dr. Maher Shakir **Dr. Hameed Aswad**

CHAPTER THREE

LEVELING—THEORY AND METHODS

General Definitions

Levelling is an operation in surveying performed to determine the difference in levels of two points. By this operation the height of a point from a *datum*, known as *elevation*, is determined.

Levelling results are used to (1) design highways, railroads, canals, sewers, water supply systems, and other facilities having grade lines that best conform to existing topography; (2) lay out construction projects according to planned elevations; (3) calculate volumes of earthwork and other materials; (4) investigate drainage characteristics of an area; (5) develop maps showing general ground configurations; and (6) study earth subsidence and crustal motion.

Vertical line: A line that follows the local direction of gravity as indicated by a plumb line.

A ***level surface*** is a curved surface that at every point is perpendicular to the local plumb line (the direction in which gravity acts).

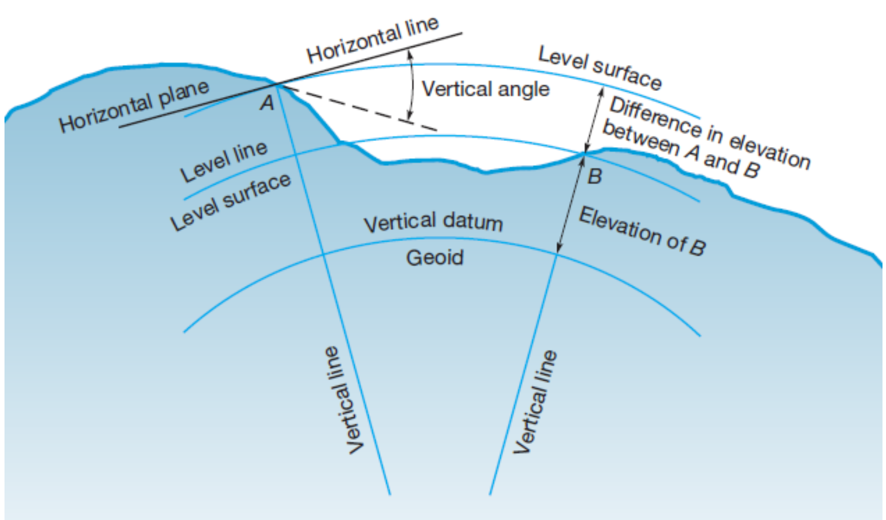
Horizontal plane. A plane perpendicular to the local direction of gravity. In plane surveying, it is a plane perpendicular to the local vertical line.

Horizontal line. A line in a horizontal plane. In plane surveying, it is a line perpendicular to the local vertical.

A ***datum*** is a reference surface of constant potential, called as a level surface of the earth's gravity field, for measuring the elevations of the points. One of such surfaces is the mean sea level surface and is considered as a standard datum. Also an arbitrary surface may be adopted as a datum.

Elevation. The distance measured along a vertical line from a vertical datum to a point or object.

Geoid. A particular level surface that serves as a datum for all elevations and astronomical observations.



Benchmark (BM). A relatively permanent object, natural or artificial, having a marked point whose elevation above or below a reference datum is known or assumed.

Height of instrument (HI), defined as the vertical distance from datum to the instrument line of sight.

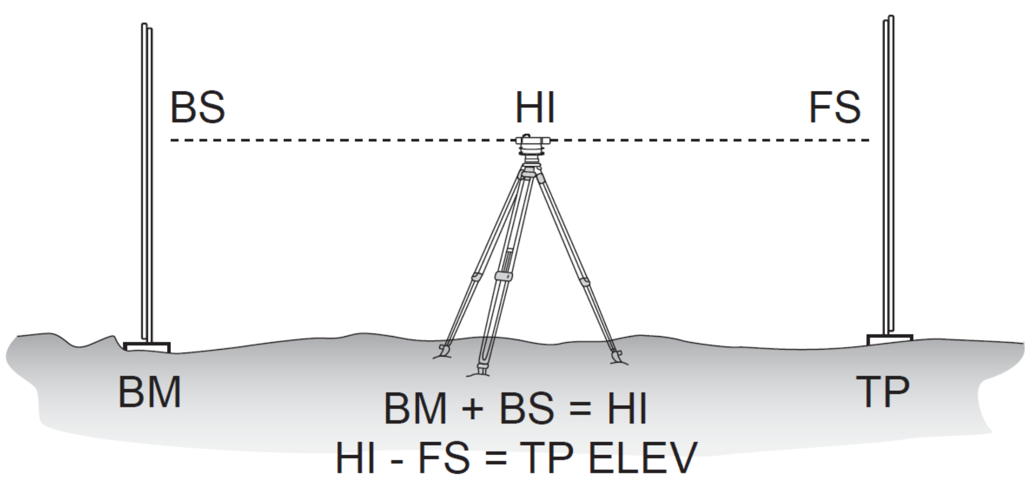
Back sight (B.S.): It is the first reading taken on the staff after setting up the level usually to determine the height of instrument. It is usually made to some form of a bench mark (B.M.) or to the points whose elevations have already been determined. When the instrument position has to be changed, the first sight taken in the next section is also a back sight.

Fore sight (F.S.): It is the last reading from an instrument position on to a staff held at a point. It is thus the last reading taken within a section of levels before shifting the instrument to the next section, and also the last reading taken over the whole series of levels.

Change point (C.P.) or turning point: A change point or turning point is the point where both the fore sight and back sight are made on a staff held at that point. A change point is required before moving the level from one section to another section. By taking the fore sight the elevation of the change point is determined and by taking the back sight the height of instrument is determined. The change points relate the various sections by making fore sight and back sight at the same point.

Intermediate sight (I.S.): The term 'intermediate sight' covers all sightings and consequent staff readings made between back sight and fore sight within each section. Thus, intermediate sight station is neither the change point nor the last point.

Rise and fall: The difference of level between two consecutive points indicates a rise or a fall between the two points. If level between A and B is positive, it is a rise and if negative, it is a fall. Rise and fall are determined for the points lying within a section.



Methods of Levelling

There are several methods for measuring vertical distances and determining the elevations of points. Traditional methods include *barometric levelling, trigonometric levelling and differential leveling.*

1. **Barometric levelling**

By using special barometers to measure air pressure (which decrease with increasing elevation), the elevation of points on the earth's surface can be determined within ±1m. This method is useful for doing a reconnaissance survey of large areas in rough country and for obtaining preliminary topographic data.

2. **Differential levelling**

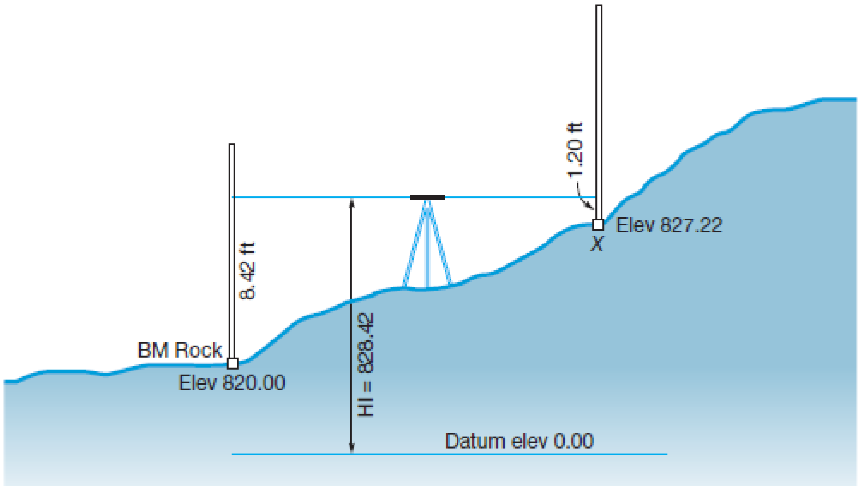
By far the most common leveling method, and the one which most surveyors are concerned with, is differential leveling. It may also be called *spirit leveling*, because the basic instrument used comprises a telescopic sight and a sensitive *spirit bubble vial.* The spirit bubble vial serves to align the telescopic sight in a horizontal direction, that is, perpendicular to the direction of gravity.

Briefly, a horizontal line of sight is first established with an instrument called a *level.* The level is securely mounted on a stand called *a tripod*, and the line of sight is made horizontal. Then the surveyor looks through the *telescopic sight* towards a *graduated level rod,* which is held vertically at a specific location or point on the ground. A reading is observed on the rod where it appears to be intercepted by the horizontal cross hair of the level; this is the vertical distance from the point on the ground up to the line of sight of the instrument.

Generally, if the elevation of point A is already known or assumed, then the rod reading on a point of known elevation is termed as a *back sight reading (plus sight,* because it must be added to the known elevation of point A to determine the elevation of the line of sight).

For example, suppose the elevation of point A is 100.00 m (above MSL), and the rod reading is 1.00m. It is clear that the elevation of the line of sight is 100.00+1.00 = 101.00m. The elevation of the horizontal line of sight through the level is called *the height of instrument* (HI).

Suppose we must determine the elevation of point B. The instrument person turns the telescope so that it faces point B, and reads the rod now held vertically on that point. For example, the rod reading might be 4.00m. A rod reading on a point of unknown elevation is called *foresight (minus sight)*. Since the HI was not changed by turning the level, we can simply subtract the foresight reading of 4.00 from the HI of 101.00 to obtain the elevation of point B, resulting here in 101.00 - 4.00 = 97.00m.



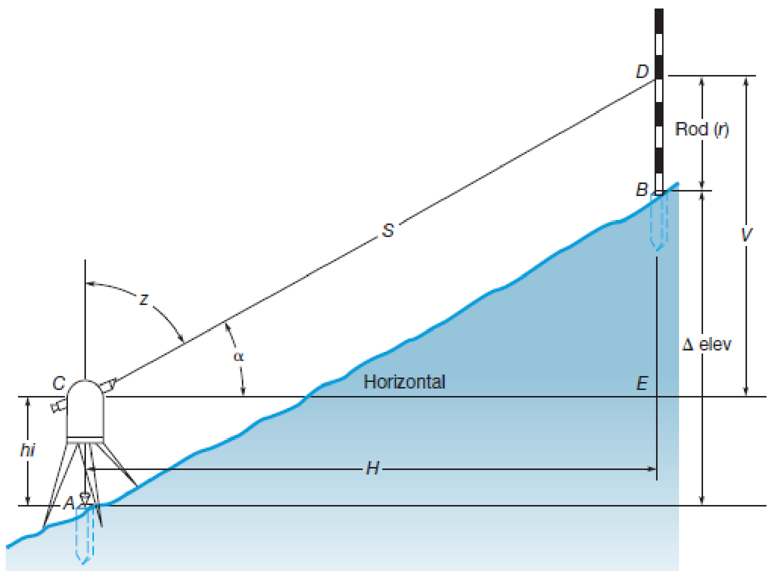
3. Trigonometric Leveling

The difference in elevation between two points can be determined by measuring (1) the inclined or horizontal distance between them and (2) the zenith angle or the altitude angle to one point from the other. (Zenith and altitude angles are measured in vertical planes. Zenith angles are observed downward from vertical, and altitude angles are observed up or down from horizontal.) Thus, in the below figure, if slope distance *S* and zenith angle *z* or altitude angle between *C* and *D* are observed, then *V*, the elevation difference between *C* and *D*, is

$$V = S \cos z$$

or

$$V = S \sin \alpha$$



Alternatively, if horizontal distance H between C and D is measured, then V is

$$V = H \cot z$$

or

$$V = H \tan \alpha$$

The difference in elevation ($\Delta elev$) between points Aand B in the figure is given by

$$\Delta elev = hi + V - r$$

where *hi* is the height of the instrument above point A and *r* the reading on the rod held at B when zenith angle z or altitude angle is read.

Levelling Equipment

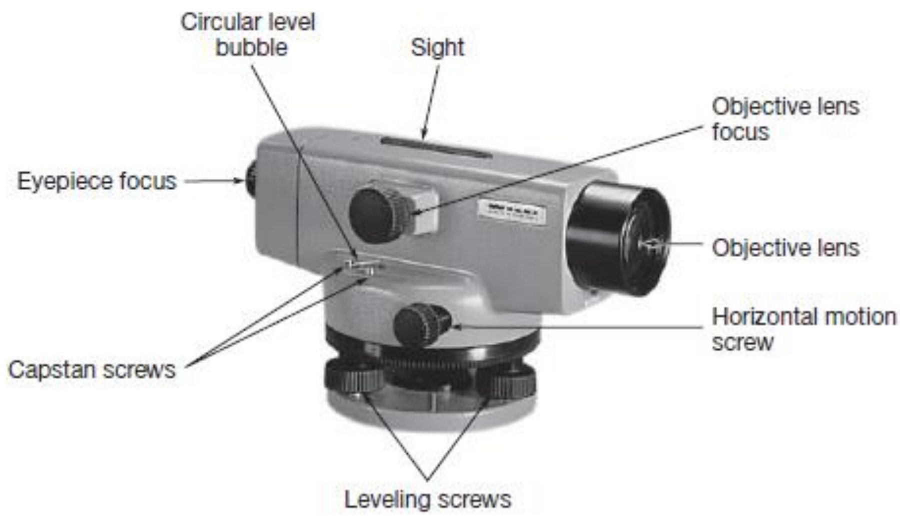
Instruments used for differential leveling can be classified into four categories: *dumpy levels*, *tilting levels*, *automatic levels*, and *digital levels*. Although each differs somewhat in design, all have two common components:

1. A telescope to create a line of sight and enable a reading to be taken on a graduated rod.
2. A system to orient the line of sight in a horizontal plane.

Dumpy and tilting levels use level vials to orient their lines of sight, while automatic levels employ automatic compensators. Digital levels also employ automatic compensators, but use bar-coded rods for automated digital readings.

TELESCOPES

The telescopes of leveling instruments define the line of sight and magnify the view of a graduated rod against a reference reticle, thereby enabling accurate readings to be obtained. The components of a telescope are mounted in a cylindrical tube. Its four main components are the objective lens, negative lens, reticle, and eyepiece. Two of these parts, the objective lens and eyepiece, are external to the instrument, and are shown on the automatic level illustrated in the below figure.



Objective Lens. This compound lens, securely mounted in the tube's object end, has its optical axis reasonably concentric with the tube axis. Its main function is to gather incoming light rays and direct them toward the negative focusing lens.

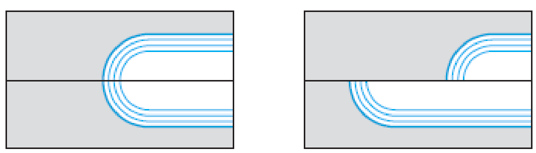
Negative Lens. The negative lens is located between the objective lens and reticle. Its function is to focus rays of light that pass through the objective lens onto the reticle plane. During focusing, the negative lens slides back and forth along the axis of the tube.

Reticle. The reticle consists in a pair of perpendicular reference lines (usually called crosshairs) mounted at the principal focus of the objective optical system. The point of intersection of the crosshairs, together with the optical center of the objective system, forms the so-called line of sight, also sometimes called the line of collimation. The crosshairs are fine lines etched on a thin round glass plate. The glass plate is held in place in the main cylindrical tube by two pairs of opposing screws, which are located at right angles to each other to facilitate adjusting the line of sight. Two additional lines parallel to and equidistant from the primary lines are commonly added to reticles for special purposes such as for three-wire levelling and for stadia.

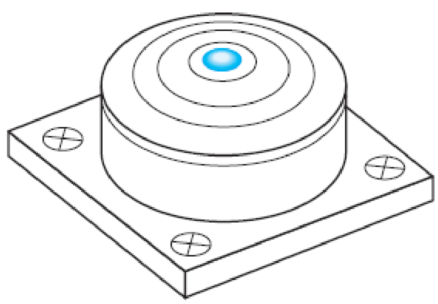
Eyepiece. The eyepiece is a microscope (usually with magnification from about 25 to 45 power) for viewing the image.

Level vial

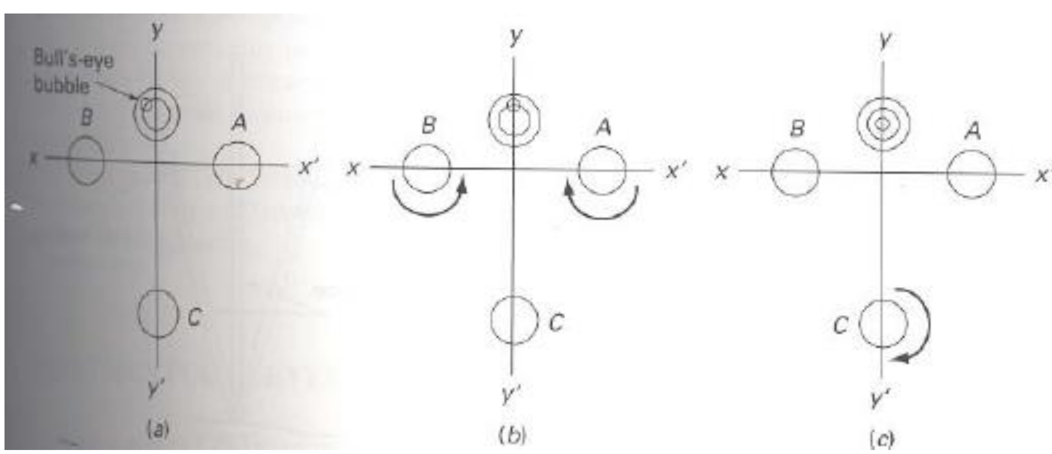
Level vials are used to orient many different surveying instruments with respect to the direction of gravity. There are two basic types: the tube vial and the circular or so-called “bull’s-eye” version. Tube vials are used on tilting levels (and also on the older dumpy levels) to precisely orient the line of sight horizontal prior to making rod readings. Bull’s-eye vials are also used on tilting levels, and on automatic levels for quick, rough leveling, after which precise final leveling occurs.



Coincidence-type level vial correctly set in left view; twice the deviation of the bubble shown in the right view.



Bull's-eye level vial.

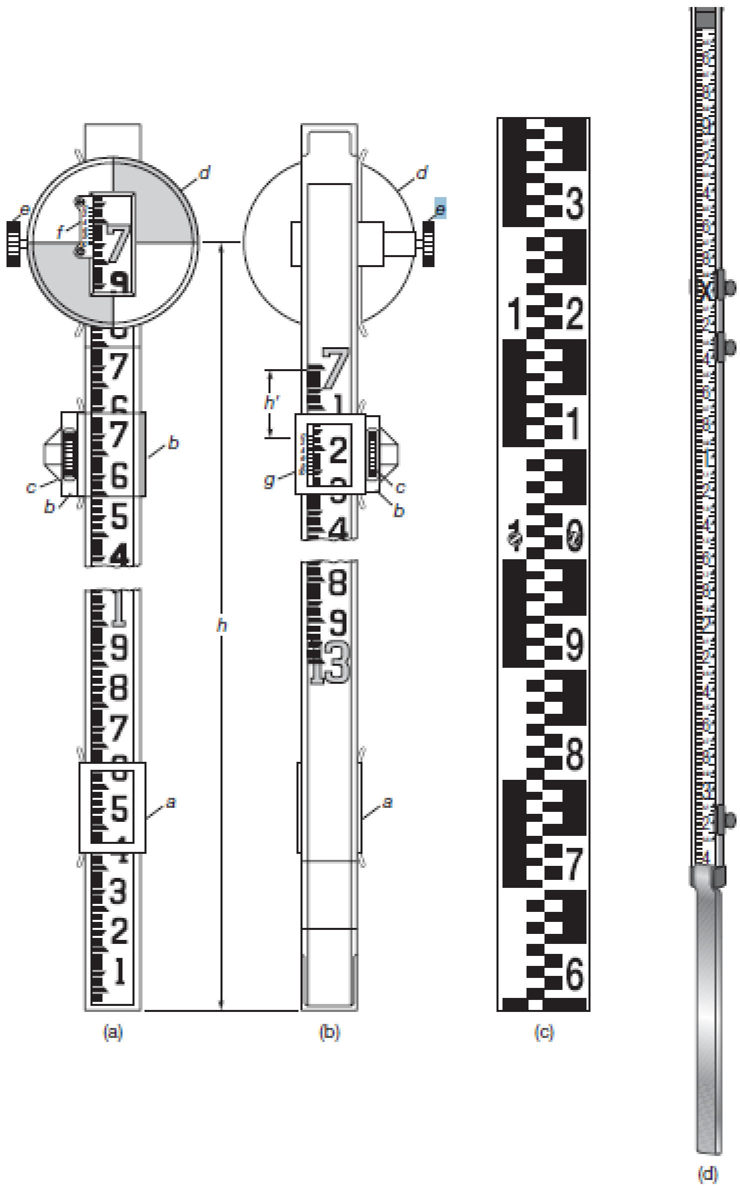


Tripods

Leveling instruments, whether tilting, automatic, or digital, are all mounted on tripods. A sturdy tripod in good condition is essential to obtain accurate results. Several types are available. The legs are made of wood or metal, may be fixed or adjustable in length, and solid or split.

Level Rods

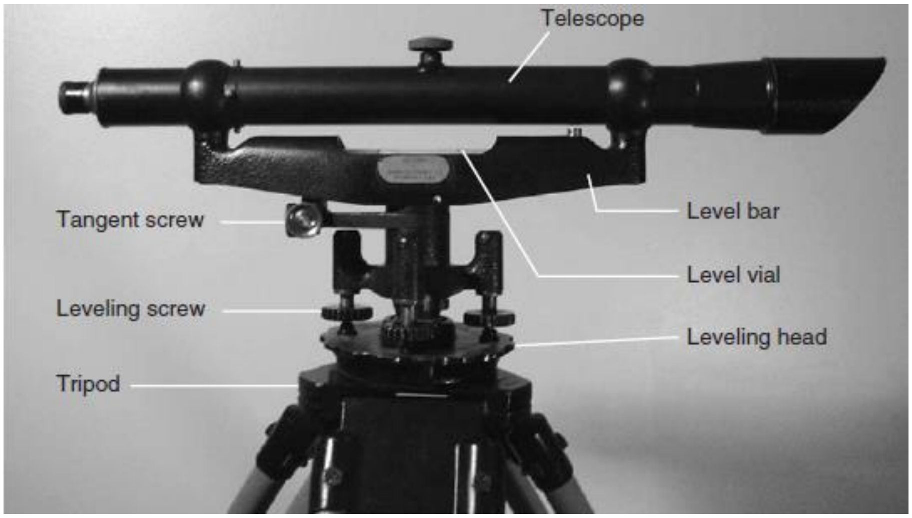
A variety of level rods are available, some of which are shown in the figure. They are made of wood, fiberglass, or metal and have graduations in feet and decimals, or meters and decimals.



(a) Philadelphia rod (front).
(b) Philadelphia rod (rear).
(c) Double-faced leveling rod with metric graduations.
(d) Lenker direct-reading rod.

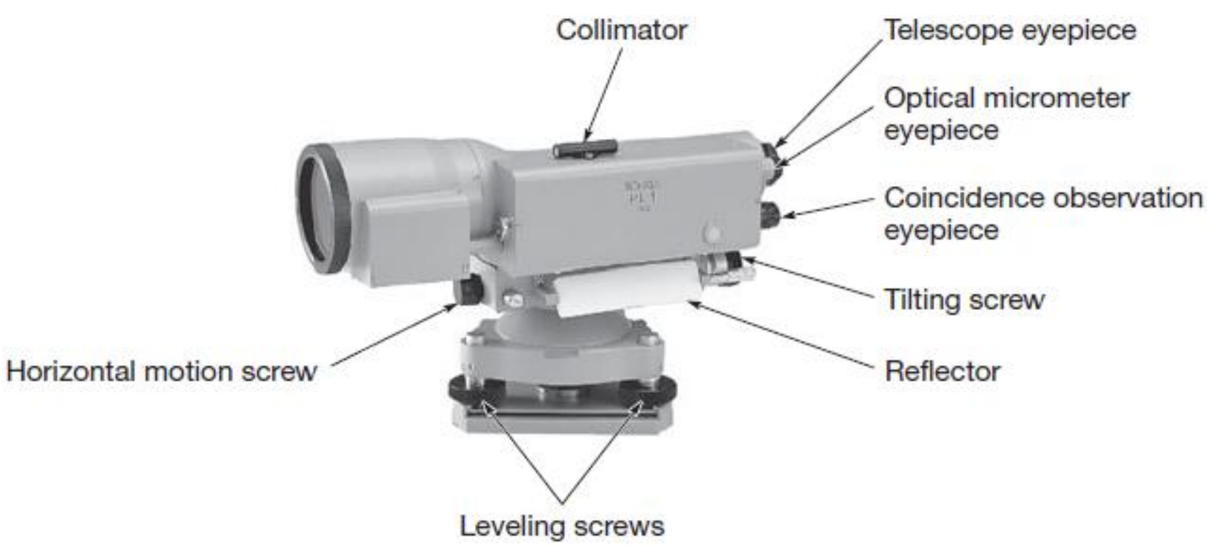
Dumpy level

Dumpy levels are rarely used today, having been replaced by other newer types.



Tilting level

Tilting levels were used for the most precise work. With these instruments, an example of which is shown in the figure, quick approximate leveling is achieved using a circular vial and the leveling screws. On some tilting levels, a ball-and socket arrangement (with no leveling screws) permits the head to be tilted and quickly locked nearly level. Precise level in preparation for readings is then obtained by carefully centering a telescope bubble. This is done for each sight, after aiming at the rod, by tilting or rotating the telescope slightly in a vertical plane about a fulcrum at the vertical axis of the instrument. A micrometer screw under the eyepiece controls this movement.



Automatic Level

Automatic levels of the type pictured in the below figure incorporate a self-leveling feature. Most of these instruments have a three-screw leveling head, which is used to quickly center a bull's-eye bubble, although some models have a ball-and-socket arrangement for this purpose. After the bull's-eye bubble is centered manually, an *automatic compensator* takes over, levels the line of sight, and keeps it level.

The system consists of prisms suspended from wires to create a pendulum. The wire lengths, support locations, and nature of the prisms are such that only horizontal rays reach the intersection of crosshairs. Thus, a horizontal line of sight is achieved even though the telescope itself may be slightly tilted away from horizontal. Damping devices shorten the time for the pendulum to come to rest, so the operator does not have to wait.

Automatic levels have become popular for general use because of the ease and rapidity of their operation. Some are precise enough for second-order and even first-order work if a parallel-plate micrometer is attached to the telescope front as an accessory. When the micrometer plate is tilted, the line of sight is displaced parallel to itself, and decimal parts of rod graduations can be read by means of a graduated dial.

Digital Level

The newest type of automatic level, the *electronic digital level*, is pictured in the figure. It is classified in the automatic category because it uses a pendulum compensator to level itself, after an operator accomplishes rough leveling with a circular bubble. With its telescope and crosshairs, the instrument could be used to obtain readings manually, just like any of the automatic levels. However, this instrument is designed to operate by employing electronic digital image processing.

After leveling the instrument, its telescope is turned toward a special bar-coded rod and focused. At the press of a button, the image of bar codes in the telescope's field of view is captured and processed. This processing consists of an on board computer comparing the captured image to the rod's entire pattern, which is stored in memory. When a match is found, which takes about 4 sec, the rod reading is displayed digitally. It can be recorded manually or automatically stored in the instrument's data collector.

TESTING AND ADJUSTING LEVELS

1. Requirements for Testing and Adjusting Instruments

To properly test and adjust leveling instruments in the field, the following rules should be followed:

1. Choose terrain that permits solid setups in a nearly level area enabling sights of at least 200 ft to be made in opposite directions.

2. Perform adjustments when good atmospheric conditions prevail, preferably on cloudy days free of heat waves. No sight line should pass through alternate sun and shadow, or be directed into the sun.

3. Place the instrument in shade, or shield it from direct rays of the sun.

4. Make sure the tripod shoes are tight and the instrument is screwed onto the tripod firmly. Spread the tripod legs well apart and position them so that the tripod plate is nearly level. Press the shoes into the ground firmly.

2. Adjusting for Parallax

The adjustment is done by carefully focusing the objective lens and eyepiece so that the crosshairs appear clear and distinct.

3. Testing and Adjusting Level Vials

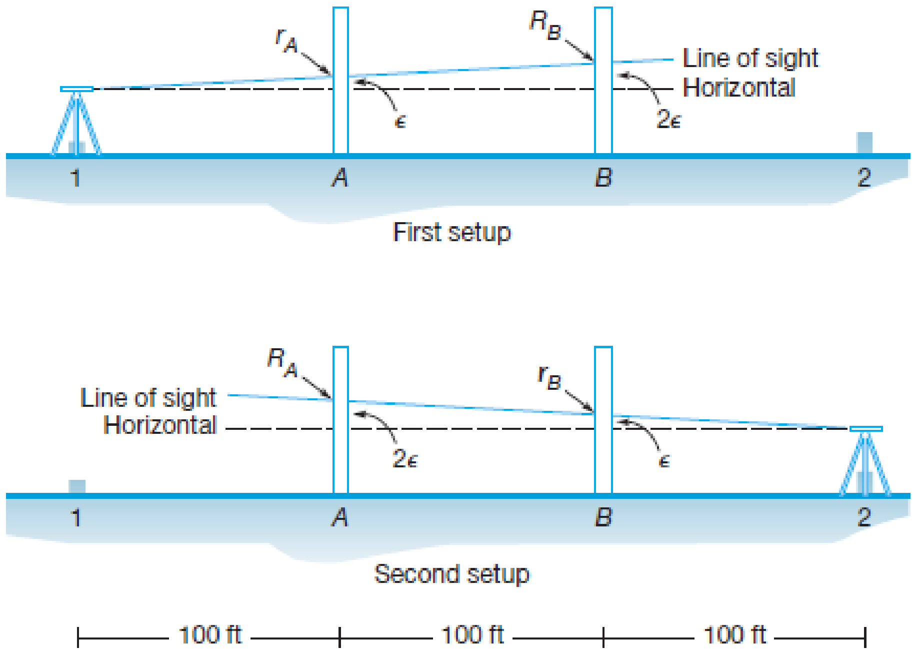
- For leveling instruments that employ a level vial, the axis of the level vial should be perpendicular to the vertical axis of the instrument. Then once the bubble is centered, the instrument can be turned about its vertical axis in any azimuth and the bubble will remain centered.
- To correct any maladjustment, turn the capstan nuts at one end of the level vial to move the bubble halfway back to the centered position. Level the instrument using the leveling screws. Repeat the test until the bubble remains centered during a complete revolution of the telescope.

Testing and Adjusting the Line of Sight

- For tilting levels, when the bubble of the level vial is centered, the line of sight should be horizontal. In other words, the axis of the level vial and the line of sight must be parallel. If they are not, a collimation error exists.
- For the automatic levels, after rough leveling by centering the circular bubble, the automatic compensator must define a horizontal line of sight if it is in proper adjustment. If it does not, the compensator is out of adjustment, and again a collimation error exists.
- The collimation error will not cause errors in differential leveling as long as backsight and foresight distances are balanced. However, it will cause errors when backsights and foresights are not balanced, which sometimes occurs in differential leveling, and cannot be avoided.
- Whether or not there is a collimation error, the difference between the rod readings at 1 should equal the difference of the two readings at 2.

$$(R_B - 2\varepsilon) - (r_A - \varepsilon) = (r_B - \varepsilon) - (R_A - 2\varepsilon)$$

$$\varepsilon = \frac{R_B - r_A - r_B + R_A}{2}$$



Example 5.1: A horizontal collimation test is performed on an automatic level following the procedures just described. With the instrument setup at point 1, the rod reading at *A* was 5.630 ft, and to *B* was 5.900 ft. After moving and leveling the instrument at point 2, the rod reading to *A* was determined to be 5.310 ft and to *B* 5.560 ft. As shown in above figure, the distance between the points was 100 ft. What is the collimation error of the instrument, and the corrected reading to *A* from point 2?

Solution

The collimation error is

$$\varepsilon = \frac{5.900 - 5.630 - 5.560 + 5.310}{2} = 0.010 \text{ ft}$$

Thus the corrected reading to A from point 2 is

$$R = 5.310 - 2(0.010) = 5.290 \text{ ft}$$

Leveling Mistakes and Errors

As with any surveying operation, blunders must be eliminated and errors minimized while running levels. Misreading the rod is a common blunder; it can be avoided by always having the rod person check the reading with pencil point or target. Note keeping mistakes can be particularly troublesome. The computations of HI and turning point (TP) elevation should be done in the field, as the work progresses. A simple arithmetic check at the end of the levelling run can be made to avoid addition or subtraction errors.

a. Random Errors

Unavoidable accidental errors may occur when running levels, for several reasons. For example:

- The level rod may not be precise when the reading is taken.
- Heat waves from the ground make it difficult to read.
- On windy day, slight vibration of the cross hair can cause small errors in the reading.
- The instrument may be slightly out of level if the spirit level is not perfectly centred.

Accidental errors can be minimized with a properly maintained and adjusted instrument if the following steps are taken:

1. Make sure the tripod legs are secure and firmly anchored before levelling the instrument.
2. Check to see that the bubble is centred before each reading; re-centre it if necessary.
3. Do not lean on the tripod legs when reading the rod.
4. Have the rod person use a rod level, to make sure it is held vertically.
5. Try to keep the line of sight about 0.5m above the ground when positioning the instrument.
6. Do not use very long BS and FS reading.

b. Systematic / Instrumental Errors

- Incorrect length of the rod.
- When the bubble tube axis is not perpendicular to the standing axis of the instrument
- When the line of sight of the telescope is not parallel to the bubble axis.

If the line of sight of a level is not exactly horizontal when the bubble is centred, but slopes either up or down, it will slope by the same amount for any direction of the telescope. As long as the horizontal lengths of the BS and FS are the same, from any given instrument position to the rod, the line of sight will intercept the rod held on each point with exactly the same error in height. But since one of the sights is a plus sight (+) and other a minus sight (-), the two errors will cancel each other out in the levelling computation.

Stadia

The stadia method determines the horizontal distance to points through the use of readings on the upper and lower (stadia) wires on the reticle.

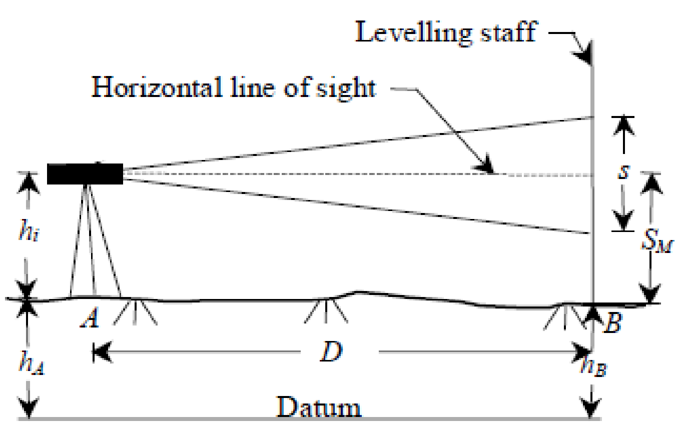
In stadia, the line of sight may be kept horizontal or inclined depending upon the field conditions. In the case of horizontal line of sight (Fig. 2.6), the horizontal distance between the instrument at A and the staff at B is

$$D = ks + c$$

Where k and c = the multiplying and additive constants, and s = the staff intercept, = $ST - SB$, where ST and SB are the top hair and bottom hair readings, respectively.

Generally, the value of k and c are kept equal to 100 and 0 (zero), respectively, for making the computations simpler. Thus

$$D = 100\ s$$

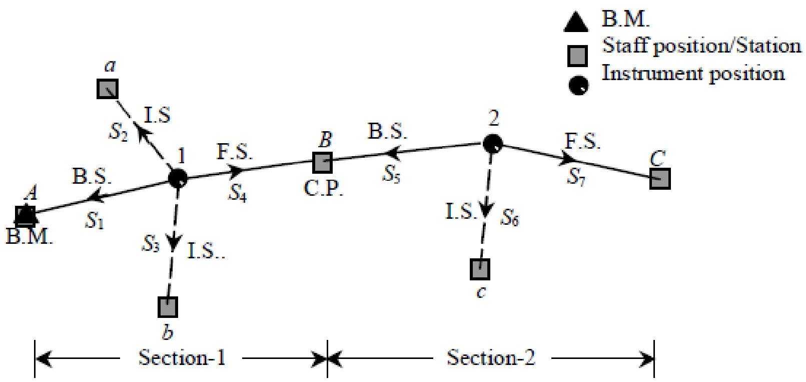


The elevations of the points, in this case, are obtained by determining the height of instrument and taking the middle hair reading. Let

hi = the height of the instrument axis above the ground at A,

h_A, h_B = the elevations of A and B, and

Section: A section comprises of one back sight, one fore sight and all the intermediate sights taken from one instrument set up within that section. Thus the number of sections is equal to the number of set ups of the instrument. (From A to B for instrument position 1 is section-1 and from B to C for instrument position 2 is section-2 in figure).



In reducing the levels for various points by the ***height of instrument method***, the height of instrument (H.I.) for the each section highlighted by different shades, is determined by adding the elevation of the point to the back sight reading taken at that point. The H.I. remains unchanged for all the staff readings taken within that section and therefore, the levels of all the points lying in that section are reduced by subtracting the corresponding staff readings, i.e., I.S. or F.S., from the H.I. of that section.

For the height of instrument method

(*i*) Σ B.S. – Σ F.S. = Last R.L. – First R.L.

(*ii*) Σ [H.I. × (No. of I.S.'s + 1)] – Σ I.S. – Σ F.S. = Σ R.L. – First R.L.

Height of instrument method

Station	B.S.	I.S.	F.S.	H.I.	R.L.	Remarks	
A	S_1			H.I.$_A$ = $h_A + S_1$	h_A	B.M. = ha	Section-1
a		S_2			h_a = H.I.$_A - S_2$		
b		S_3			h_b = H.I.$_A - S_3$		
B	S_5		S_4	H.I.$_B$ = $h_B + S_5$	h_B = H.I.$_A - S_4$	C.P.	–
c		S_6			h_c = H.I.$_B - S_6$		Section-2
C			S_7		H_C = H.I.$_B - S_7$		
	Σ B.S.		Σ F.S.				
Check: Σ B.S. – Σ F.S. = Last R.L. – First R.L.							

In the ***rise and fall method***, the rises and the falls are found out for the points lying within each section. Adding or subtracting the rise or fall to or from the reduced level of the backward station obtains the level for a forward station. In the table, *r* and *f* indicate the rise and the fall, respectively, assumed between the consecutive points.

For the rise and fall method

$$\Sigma \text{ B.S.} - \Sigma \text{ F.S.} = \Sigma \text{ Rise} - \Sigma \text{ Fall} = \text{Last R.L.} - \text{First R.L.}$$

Rise and fall method

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks	
A	S_1					h_A	B.M. = h_a	Section-1
a		S_2		$r_1 = S_1 - S_2$		$h_a = h_A + r_1$		
b		S_3			$f_1 = S_2 - S_3$	$h_b = h_a - f_1$		
B	S_5		S_4		$f_2 = S_3 - S_4$	$h_B = h_b - f_2$	C.P.	
c		S_6			$f_3 = S_5 - S_6$	$h_c = h_B - f_3$		
C			S_7	$r_2 = S_6 - S_7$		$H_C = h_c + r_2$		Section-2
	Σ B.S.		Σ F.S.	Σ Rise	Σ Fall			
Check:	Σ B.S. − Σ F.S. = Σ Rise − Σ Fall = Last R.L. − First R.L.							

Loop Closure and Its Apportioning

A *loop closure* or *misclosure* is the amount by which a level circuit fails to close. It is the difference of elevation of the measured or computed elevation and known or established elevation of the same point. Thus loop closure is given by

e = computed value of R.L. – known value of R.L.

If the length of the loop or circuit is L and the distance of a station to which the correction c is computed, is l, then

$$c = -\frac{l}{L}$$

Alternatively, the correction is applied to the elevations of each change point and the closing point of known elevation. If there are n_1 change points then the total number points at which the corrections are to be applied is

$$n = n_1 + 1$$

and the correction at each point is

$$= -\frac{e}{n}$$

The corrections at the intermediate points are taken as same as that for the change points to which they are related.

Another approach could be to apply total of $-e/2$ correction equally to all the back sights and total of $+e/2$ correction equally to all the fore sights. Thus if there are n_B back sights and n_F fore sights then

$$\text{correction to each back sight} = -\frac{e}{2n_B}$$

$$\text{correction to each fore sight} = +\frac{e}{2n_F}$$

Profile Leveling

Before engineers can properly design linear facilities such as highways, railroads, transmission lines, aqueducts, canals, sewers, and water mains, they need accurate information about the topography along the proposed routes. Profile leveling, which yields elevations at definite points along a reference line, provides the needed data.

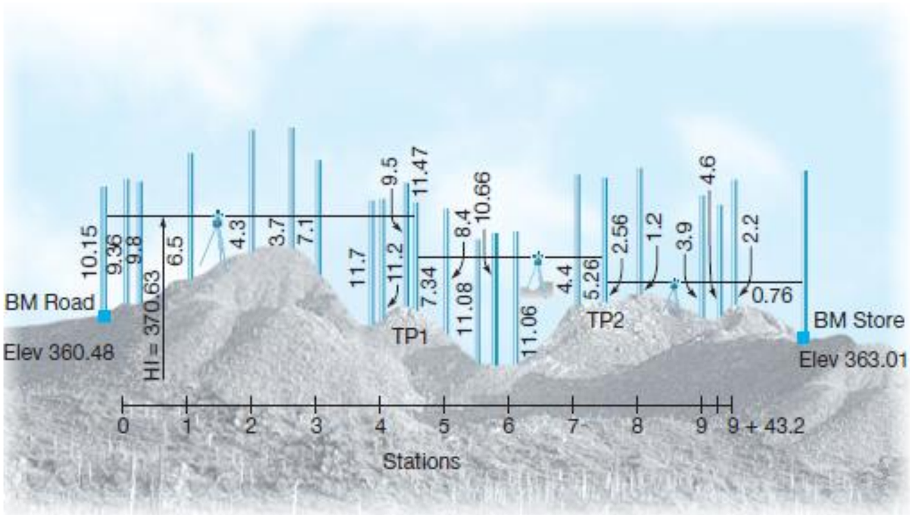
Staking and Stationing the Reference Line

Depending on the particular project, the reference line may be a single straight segment, as in the case of a short sewer line; a series of connected straight segments which change direction at angle points, as with transmission lines; or straight segments joined by curves, which occur with highways and railroads. The required alignment for any proposed facility will normally have been selected as the result of a preliminary design, which is usually based on a study of existing maps and aerial photos. The reference alignment will most often be the proposed construction centerline, although frequently offset reference lines are used.

To stake the proposed reference line, key points such as the starting and ending points and angle points will be set first. Then intermediate stakes will be placed on line, usually at 10-, 20-, 30-, or 40-m spacing, depending on conditions.

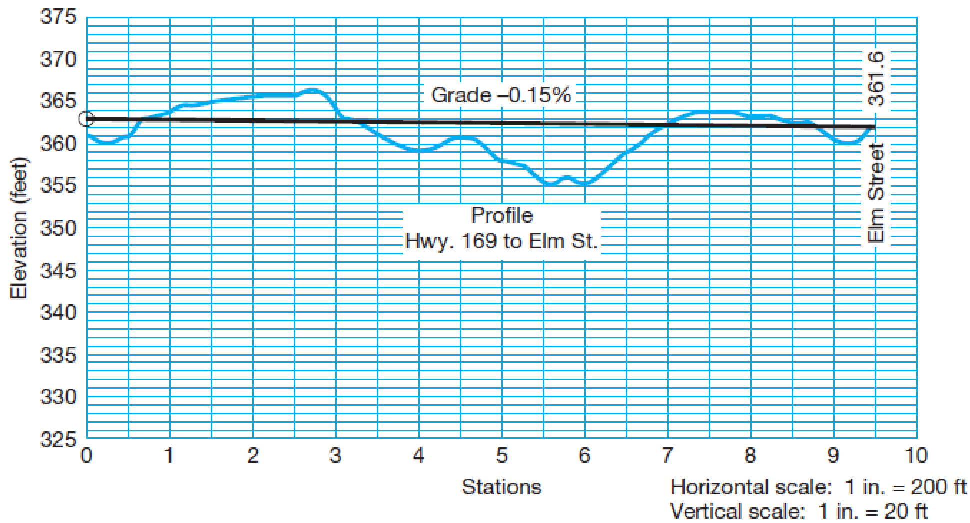
In route surveying, a system called ***stationing*** is used to specify the relative horizontal position of any point along the reference line. If the metric system is used, full stations are 1 km (1000 m) apart. The starting point of a reference line might be arbitrarily designated as 1+000 or 10+00 but again 0+000 could be used.

- Profile leveling consists simply of differential leveling with the addition of intermediate minus sights (foresights) taken at required points along the reference line.



Drawing and Using the Profile

- Prior to drawing the profile, it is first necessary to compute elevations along the reference line from the field notes.
- An adjustment has been made to distribute any misclosure in the level circuit.
- The profile is then drawn by plotting elevations on the ordinate versus their corresponding stations on the abscissa. By connecting adjacent plotted points, the profile is realized.
- Until recently, profiles were manually plotted, usually on special paper like the type shown in the figure.
- The rate of grade (or gradient or percent grade) is the rise or fall in feet per 100 ft, or in meters per 100 m. Ascending grades are plus; descending grades, minus. A gradeline of 0.15%, chosen to approximately equalize cuts and fills. Along this grade line, elevations drop at the rate of 0.15 ft per 100 ft.



Grid, Cross-Section, or Borrow-Pit Leveling

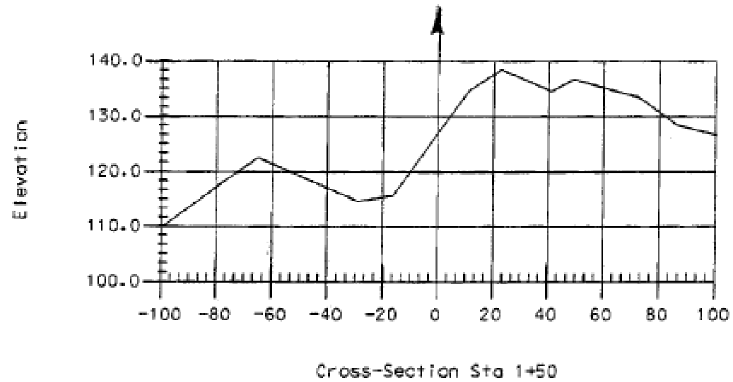
Grid leveling is a method for locating contours. It is accomplished by staking an area in squares of 10, 20, 50, 100, or more feet (or comparable meter lengths) and determining the corner elevations by differential leveling.

Cross sections are lines 90 degrees perpendicular to the alignment (centerline), along which the configuration of the ground is determined by obtaining elevations of points at known distances from the alignment.

Cross sections are used to determine the shape of the ground surface through the alignment corridor. The shape of the ground surface helps the designer pick his horizontal and vertical profile. Once the alignment is picked, earthwork quantities can be calculated. The earthwork quantities will then be used to help evaluate the alignment choice.

In addition to earthwork calculations, cross sections are used in the design of storm sewers, culvert extensions and the size and location of new culverts. Because of this fact it becomes more important to get the additional sections at the points of interest that do not fall on the 50 foot stations.

The traditional method of taking cross sections starts with an alignment staked out in the field. A profile is run over the centerline stations by differential leveling. Cross section lines are laid out 90 degrees to the alignment, often with a right angle prism. Usually elevations are determined with an engineer’s level and rod in level terrain or with a hand level and rod in rough, irregular country. For each cross section, the height of instrument is determined by a backsight on the centerline station. The rod is then held on the cross section line at breaks in the surface slope, where rod readings are observed and distances measured with a tape. Cross sections are usually taken at even stations and points of interest or irregularity along the alignment.



Example 5.2: The following readings were taken with a level and 4 m staff. Draw up a level book page and reduce the levels by the height of instrument method.

0.578 B.M. (= 58.250 m), 0.933, 1.768, 2.450, (2.005 and 0.567) C.P., 1.888, 1.181, (3.679 and 0.612) C.P., 0.705, 1.810.

Solution:

The first reading being on a B.M., is a back sight. As the fifth station is a change point, 2.005 is fore sight reading and 0.567 is back sight reading. All the readings between the first and fifth readings are intermediate sight-readings. Similarly, the eighth station being a change point, 3.679 is fore sight reading, 0.612 is back sight reading, and 1.888, 1.181 are intermediate sight readings. The last reading 1.810 is fore sight and 0.705 is intermediate sight-readings. All the readings have been entered in their respective columns in the following table and the levels have been reduced by height of instrument method. In the following computations, the values of B.S., I.S., H.I., etc., for a particular station have been indicated by its number or name.

$$\begin{aligned}
\text{H.I.}_1 &= h_1 + \text{B.S.}_1 = 58.250 + 0.578 = 58.828 \text{ m}\\
h_2 &= \text{H.I.}_1 - \text{I.S.}_2 = 58.828 - 0.933 = 57.895 \text{ m}\\
h_3 &= \text{H.I.}_1 - \text{I.S.}_3 = 58.828 - 1.768 = 57.060 \text{ m}\\
h_4 &= \text{H.I.}_1 - \text{I.S.}_4 = 58.828 - 2.450 = 56.378 \text{ m}\\
h_5 &= \text{H.I.}_1 - \text{F.S.}_5 = 58.828 - 2.005 = 56.823 \text{ m}
\end{aligned}$$

$$\text{H.I.}_5 = h_5 + \text{B.S.}_5 = 56.823 + 0.567 = 57.390 \text{ m}$$
$$h_6 = \text{H.I.}_2 - \text{I.S.}_6 = 57.390 - 1.888 = 55.502 \text{ m}$$
$$h_7 = \text{H.I.}_2 - \text{I.S.}_7 = 57.390 - 1.181 = 56.209 \text{ m}$$
$$h_8 = \text{H.I.}_2 - \text{F.S.}_8 = 57.390 - 3.679 = 53.711 \text{ m}$$

$$\text{H.I.}_8 = h_8 + \text{B.S.}_8 = 53.711 + 0.612 = 54.323 \text{ m}$$
$$h_9 = \text{H.I.}_8 - \text{I.S.}_9 = 54.323 - 0.705 = 53.618 \text{ m}$$
$$h_{10} = \text{H.I.}_8 - \text{F.S.}_{10} = 54.323 - 1.810 = 52.513 \text{ m}$$

Additional check for H.I. method: Σ [H.I. $\times$ (No. of I.S.s + 1)] – Σ I.S. – Σ F.S. = Σ R.L. – First R.L.

[58.828 × 4 + 57.390 × 3 + 54.323 × 2] – 8.925 – 7.494 = 557.959 – 58.250 = 499.709 (*O.K.*)

Station	B.S.	I.S.	F.S.	H.I.	R.L.	Remarks
1	0.578			58.828	58.250	B.M.=58.250 m
2		0.933			57.895	
3		1.768			57.060	
4		2.450			56.378	
5	0.567		2.005	57.390	56.823	C.P.
6		1.888			55.502	
7		1.181			56.209	
8	0.612		3.679	54.323	53.711	C.P.
9		0.705			53.618	
10			1.810		52.513	
Σ	1.757	8.925	7.494		557.956	
Check: 1.757 – 7.494 = 52.513 – 58.250 = – 5.737 (*O.K.*)						

Example 5.3: Reduce the levels of the stations from the readings given in the Example 5.2 by the rise and fall method.

Solution:

Section-1

$$f_2 = \text{B.S.}_1 - \text{I.S.}_2 = 0.578 - 0.933 = 0.355$$
$$f_3 = \text{I.S.}_2 - \text{I.S.}_3 = 0.933 - 1.768 = 0.835$$
$$f_4 = \text{I.S.}_3 - \text{I.S.}_4 = 1.768 - 2.450 = 0.682$$
$$r_5 = \text{I.S.}_4 - \text{F.S.}_5 = 2.450 - 2.005 = 0.445$$

Section-2: $f_6 = \text{B.S.}_5 - \text{I.S.}_6 = 0.567 - 1.888 = 1.321$

$f_7 = \text{I.S.}_6 - \text{I.S.}_7 = 1.888 - 1.181 = 0.707$

$f_8 = \text{I.S.}_7 - \text{F.S.}_8 = 1.181 - 3.679 = 2.498$

Section-3: $f_9 = \text{B.S.}_8 - \text{I.S.}_9 = 0.612 - 0.705 = 0.093$

$f_{10} = \text{I.S.}_9 - \text{F.S.}_{10} = 0.705 - 1.810 = 1.105$

Calculation of reduced levels

$h_2 = h_1 - f_2 = 58.250 - 0.355 = 57.895$ m

$h_3 = h_2 - f_3 = 57.895 - 0.835 = 57.060$ m

$h_4 = h_3 - f_4 = 57.060 - 0.682 = 56.378$ m

$h_5 = h_4 + r_5 = 56.378 + 0.445 = 56.823$ m

$h_6 = h_5 - f_6 = 56.823 - 1.321 = 55.502$ m

$h_7 = h_6 + r_7 = 55.502 + 0.707 = 56.209$ m

$h_8 = h_7 - f_8 = 56.209 - 2.498 = 53.711$ m

$h_9 = h_8 - f_9 = 53.711 - 0.093 = 53.618$ m

$h_{10} = h_9 - f_{10} = 53.618 - 1.105 = 52.513$ m

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	0.578					58.250	B.M.=58.250 m
2		0.933			0.355	57.895	
3		1.768			0.835	57.060	
4		2.450			0.682	56.378	
5	0.567		2.005	0.445		56.823	C.P.
6		1.888			1.321	55.502	
7		1.181		0.707		56.209	
8	0.612		3.679		2.498	53.711	C.P.
9		0.705			0.093	53.618	
10			1.810		1.105	52.513	
Σ	1.757		7.494	1.152	6.889		
Check: 1.757 – 7.494 = 1.152 – 6.889 = 52.513 – 58.250 = – 5.737 (*O.K.*)							

Example 5.4: A page of level book is reproduced below in which some readings marked as (X), are missing. Complete the page with all arithmetic checks.

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	3.150					×	
2	1.770		×		0.700	×	C.P.
3		2.200			×	×	
4	×		1.850	×		×	C.P.
5		2.440			0.010	×	
6	×		×	1.100		×	C.P.
7	1.185		2.010	×		222.200	C.P.
8		–2.735		×		×	Staff held inverted
9	×		1.685		4.420	×	C.P.
10			1.525		0.805	×	
Σ	12.055		×	×	×		

Solution:

The computations of the missing values are explained below.

$$B.S._4 - I.S._5 = f_5, \quad B.S._4 = f_5 + I.S._5 = -0.010 + 2.440 = \mathbf{2.430}$$

$$B.S._9 - F.S._{10} = f_{10}, \quad B.S._9 = f_{10} + F.S._{10} = -0.805 + 1.525 = \mathbf{0.720}$$

$$B.S._1 + B.S._2 + B.S._4 + B.S._6 + B.S._7 + B.S._9 = \Sigma B.S.$$

$$3.150 + 1.770 + 2.430 + B.S._6 + 1.185 + 0.720 = 12.055$$

$$B.S._6 = 12.055 - 9.255 = \mathbf{2.800}$$

$$B.S._1 - F.S._2 = f_2, \quad F.S._2 = B.S._1 - f_2 = 3.150 - (-0.700) = \mathbf{3.850}$$

$$I.S._5 - F.S._6 = r_6, \quad F.S._6 = I.S._5 - r_6 = 2.440 - 1.100 = \mathbf{1.340}$$

$$B.S._2 - I.S._3 = 1.770 - 2.200 = -0.430 = \mathbf{0.430}\ (\textit{fall}) = f_3$$

$$I.S._3 - F.S._4 = 2.200 - 1.850 = \mathbf{0.350} = r_4$$

$$B.S._6 - F.S._7 = 2.800 - 2.010 = \mathbf{0.790} = r_7$$

$$B.S._7 - I.S._8 = 1.185 - (-2.735) = \mathbf{3.920} = r_8$$

$$h_6 = h_7 - r_7 = 222.200 - 0.790 = 221.410 \text{ m}$$
$$h_5 = h_6 - r_6 = 221.410 - 1.100 = 220.310 \text{ m}$$
$$h_4 = h_5 + f_5 = 220.310 + 0.010 = 220.320 \text{ m}$$
$$h_3 = h_4 - r_4 = 220.320 - 0.350 = 219.970 \text{ m}$$
$$h_2 = h_3 + f_3 = 219.970 + 0.430 = 220.400 \text{ m}$$
$$h_1 = h_2 + f_2 = 220.400 + 0.700 = 221.100 \text{ m}$$
$$h_8 = h_7 + r_8 = 222.200 + 3.920 = 226.120 \text{ m}$$
$$h_9 = h_8 - f_9 = 226.120 - 4.420 = 221.700 \text{ m}$$
$$h_{10} = h_9 - f_{10} = 221.700 - 0.805 = 220.895 \text{ m}$$

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	3.150					221.100	
2	1.770		3.850		0.700	220.400	C.P.
3		2.200			0.430	219.970	
4	2.430		1.850	0.350		220.320	C.P.
5		2.440			0.010	220.310	
6	2.800		1.340	1.100		221.410	C.P.
7	1.185		2.010	0.790		222.200	C.P.
8		-2.735		3.920		226.120	Staff held inverted
9	0.720		1.685		4.420	221.700	C.P.
10			1.525		0.805	220.895	
Σ	12.055		12.266	6.610	6.365		
Check: 12.055 – 12.266 = 6.610 – 6.365 = 220.895 – 221.100 = – 0.205 (*O.K.*)							

Example 5.5: The readings given in the table, were recorded in a levelling operation from points 1 to 10. Reduce the levels by the height of instrument method and apply appropriate checks. The point 10 is a bench mark having elevation of 66.374 m. Determine the loop closure and adjust the calculated values of the levels by applying necessary corrections. Also determine the mean gradient between the points 1 to 10.

Station	Chainage (m)	B.S.	I.S.	F.S.	Remarks
1	0	0.597			B.M.= 68.233 m
2	20	2.587		3.132	C.P
3	40		1.565		
4	60		1.911		
5	80		0.376		
6	100	2.244		1.522	C.P
7	120		3.771		
8	140	1.334		1.985	C.P
9	160		0.601		
10	180			2.002	

Solution:

$$\begin{aligned}
H.I._{1} &= h_1 + B.S._{1} = 68.233 + 0.597 = 68.830 \text{ m} \\
h_2 &= H.I._{1} - F.S._{2} = 68.233 + 3.132 = 65.698 \text{ m} \\
H.I._{2} &= h_2 + B.S._{2} = 65.698 + 2.587 = 68.285 \text{ m} \\
h_3 &= H.I._{2} - I.S._{3} = 68.285 - 1.565 = 66.720 \text{ m} \\
h_4 &= H.I._{2} - I.S._{4} = 68.285 - 1.911 = 66.374 \text{ m} \\
h_5 &= H.I._{2} - I.S._{5} = 68.285 - 0.376 = 67.909 \text{ m} \\
h_6 &= H.I._{2} - F.S._{6} = 68.285 - 1.522 = 66.763 \text{ m} \\
H.I._{6} &= h_6 + B.S._{6} = 66.763 + 2.244 = 69.007 \text{ m} \\
h_7 &= H.I._{6} - I.S._{7} = 69.007 - 3.771 = 65.236 \text{ m} \\
h_8 &= H.I._{6} - F.S._{8} = 69.007 - 1.985 = 67.022 \text{ m} \\
H.I._{8} &= h_8 + B.S._{8} = 67.022 + 1.334 = 68.356 \text{ m} \\
h_9 &= H.I._{8} - I.S._{9} = 68.356 - 0.601 = 67.755 \text{ m} \\
h_{10} &= H.I._{8} - F.S._{10} = 68.356 - 2.002 = 66.354 \text{ m}
\end{aligned}$$

Loop closure and loop adjustment

The error at point 10 = computed R.L. – known R.L. = 66.354 – 66.374 = –0.020 m

Therefore correction = +0.020 m

Since there are three change points, there will be four instrument positions. Thus the total number of points at which the corrections are to be applied is four, i.e., three C.P.s and one last F.S. It is reasonable to assume that similar errors have occurred at each station. Therefore, the correction for each instrument setting which has to be applied progressively, is

$$= +\frac{0.020}{4} = 0.005\,\text{m}$$

the correction at station 1	0.0 m
the correction at station 2	+ 0.005 m
the correction at station 6	+ 0.010 m
the correction at station 8	+ 0.015 m
the correction at station 10	+ 0.020 m

The corrections for the intermediate sights will be same as the corrections for that instrument stations to which they are related. Therefore,

$$\text{correction for I.S.}_3,\ \text{I.S.}_4,\ \text{and I.S.}_5 = +\,0.010\ \text{m}$$
$$\text{correction for I.S.}_7 = +\,0.015\ \text{m}$$
$$\text{correction for I.S.}_9 = +\,0.020\ \text{m}$$

Applying the above corrections to the respective reduced levels, the corrected reduced levels are obtained. The results have been presented in the table:

Station	Chainage (m)	B.S.	I.S.	F.S.	H.I.	R.L.	Correction	Corrected R.L.
1	0	0.597			68.830	68.233	–	68.233
2	20	2.587		3.132	68.285	65.698	+ 0.005	65.703
3	40		1.565			66.720	+ 0.010	66.730
4	60		1.911			66.374	+ 0.010	66.384
5	80		0.376			67.909	+ 0.010	67.919
6	100	2.244		1.522	69.007	66.763	+ 0.010	66.773
7	120		3.771			65.236	+ 0.015	65.251
8	140	1.334		1.985	68.356	67.022	+ 0.015	67.037
9	160		0.601			67.755	+ 0.020	67.775
10	180			2.002		66.354	+ 0.020	66.374
Σ		6.762		8.641				
Check: 6.762 – 8.641 = 66.354 – 68.233 = – 1.879 (*O.K.*)								

Example 5.6: Determine the corrected reduced levels of the points given in Example 5.6 by two alternative methods.

Solution: Method 1

The correction $c = -\frac{l}{L}$

The total correction at point 10 (from Example 5.6) = + 0.020 m

The distance between the points 1 and 10 = 180 m

$$\text{Correction at point 2} = +\frac{0.020}{180}\times 20 = +\ 0.002\ \text{m}$$

$$\text{Correction at point 6} = +\frac{0.020}{180}\times 100 = +\ 0.011\ \text{m}$$

$$\text{Correction at point 8} = +\frac{0.020}{180}\times 140 = +\ 0.016\ \text{m}$$

$$\text{Correction at point 10} = +\frac{0.020}{180}\times 180 = +\ 0.020\ \text{m}$$

$$\text{Corrections at points 3, 4, and 5} = +\ 0.011\ \text{m}$$

$$\text{Correction at point 7} = +\ 0.016\ \text{m}$$

$$\text{Correction at point 9} = +\ 0.020\ \text{m}$$

Station	R.L.	Correction	Corrected R.L.
1	68.233	–	68.233
2	65.698	+ 0.002	65.700
3	66.720	+ 0.011	66.731
4	66.374	+ 0.011	66.385
5	67.909	+ 0.011	67.920
6	66.763	+ 0.011	66.774
7	65.236	+ 0.016	65.252
8	67.022	+ 0.016	67.038
9	67.755	+ 0.020	67.775
10	66.354	+ 0.020	66.374

Method 2

In this method half of the total correction is applied negatively to all the back sights and half of the total correction is applied positively to all the fore sights.

Total number of back sights = 4

Total number of fore sights = 4

$$\text{Correction to each back sight} = -\left(\frac{-0.020}{2\times 4}\right) = +\ 0.0025\ \text{m}$$

$$\text{Correction to each fore sight} = +\left(\frac{-0.020}{2\times 4}\right) = -\ 0.0025\ \text{m}$$

Station	Observed		Correction	Corrected				H.I.	Corrected
	B.S.	I.S.		F.S.	B.S.	I.S.	F.S.		R.L.
1	0.597	+ 0.0025	0.5995			68.8325	68.233		
2	2.587	3.132	+ 0.0025	– 0.0025	2.5895		3.1295	68.2925	65.703
3		1.565		– 0.0025		1.5625			66.730
4		1.911		– 0.0025		1.9085			66.384
5		0.376		– 0.0025		0.3735			67.919
6	2.244	1.522	+ 0.0025	– 0.0025	0.2465		1.5195	69.0195	66.773
7		3.771		– 0.0025		3.7685			65.251
8	1.334	1.985	+ 0.0025	– 0.0025	1.3365		1.9825	68.3735	67.037
9		0.601		– 0.0025		0.5985			67.775
10			2.002	– 0.0025			1.9995		66.374

University of Anbar

College of Engineering

Civil Engineering Department

LECTURE NOTE
ENGINEERING SURVEYING

Chapter Four

By

Dr. Maher Shakir **Dr. Hameed Aswad**

CHAPTER FOUR

Distance Measurements Using Trigonometric & EDM

Tacheometric Or Optical Method

In stadia tacheometry the line of sight of the tacheometer may be kept horizontal or inclined depending upon the field conditions. In the case of horizontal line of sight (Fig. 4.1), the horizontal distance between the instrument at ***A*** and the staff at ***B*** is

$$D = ks + c$$

where k and c = the multiplying and additive constants of the tacheometer, and s = the staff intercept, $= ST - SB$, where ST and SB are the top hair and bottom hair readings, respectively.

Generally, the value of k and c are kept equal to 100 and 0 (zero), respectively, for making the computations simpler. Thus

$$D = 100\,s$$

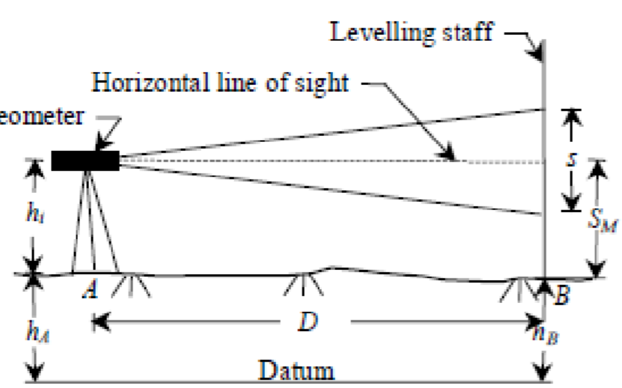


Figure 4.1

The elevations of the points, in this case, are obtained by determining the height of instrument and taking the middle hair reading. Let *hi* = the height of the instrument axis above the ground at A, *hA*, *hB* = the elevations of A and B, and *SM* = the middle hair reading then, the height of instrument is:

H.I. = hA + hi

and

hB = H.I. – SM = hA + hi – SM

In the case of inclined line of sight as shown in Fig. 4.2, the vertical angle α is measured, and the horizontal and vertical distances, *D* and *V*, respectively, are determined from the following expressions.

$$D = ks\ cos^2\alpha$$

$$V = \frac{1}{2} ks\ \sin 2\alpha$$

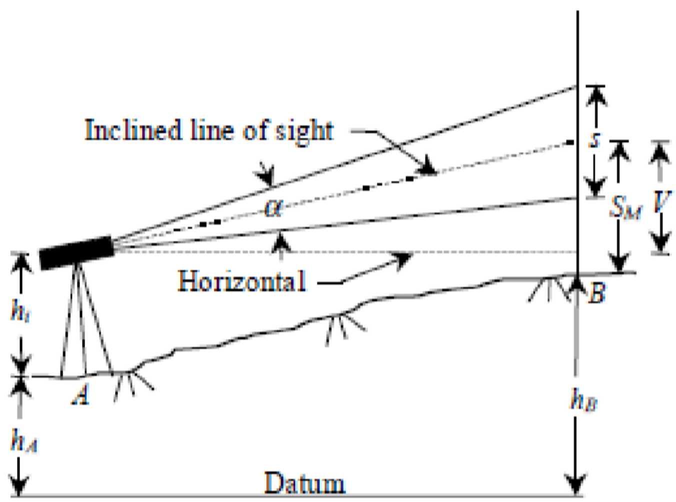


Figure 4.2

The elevation of B is computed as below.

$$hB = hA + hi + V – SM$$

Example 4.1. The following tacheometric observations were made on two points P and Q from station A.

Staff at	Vertical angle	Staff reading		
		Upper	Middle	Lower
P	$-5°12'$	1.388	0.978	0.610
Q	$+27°35'$	1.604	1.286	0.997

The height of the tacheometer at A above the ground was 1.55 m. Determine the elevations of P and Q if the elevation of A is 75.500 m. The stadia constant k and c are respectively 100 and 0.00 m.

Solution:

Since the vertical angles are given, the line of sights are inclined for both the points. From Eqs. we have

$$H = ks\cos^2\alpha$$

$$V = \frac{1}{2}ks\sin 2\alpha$$

The given data are

$$s_1 = (1.388 - 0.610) = 0.778\,\text{m}, \quad \alpha_1 = 5°12'$$

$$s_2 = (1.604 - 0.997) = 0.607\,\text{m}, \quad \alpha_2 = 27°35'$$

Therefore the distances

$$H_{AP} = 100\times 0.778\times\cos^2(5°12') = 77.161\,\text{m}$$

$$V_{AP} = \frac{1}{2}\times 100\times 0.778\times\sin(2\times 5°12') = 7.022\,\text{m}$$

$$H_{AQ} = 100\times 0.607\times\cos^2(27°35') = 47.686\,\text{m}$$

$$V_{AQ} = \frac{1}{2}\times 100\times 0.607\times\sin(2\times 27°35') = 24.912\,\text{m}$$

The height of the instrument

$$\text{H.I.} = \text{Elevation of A} + \text{instrument height}$$
$$= 75.500 + 1.55 = 77.050 \text{ m}$$

Elevation of P

$$h_P = \text{H.I.} - V_{AP} - \text{middle hair reading at } P$$
$$= 77.050 - 7.022 - 0.978$$
$$\mathbf{= 69.050\ m}$$

Elevation Q

$$H_Q = \text{H.I.} + V_{AQ} - \text{middle hair reading at } Q$$
$$= 77.050 + 24.912 - 1.286$$
$$\mathbf{= 100.676\ m.}$$

4.2 **Electronic Distance Measurement**

4.2.1 Introduction

A major advance in surveying instrumentation occurred approximately 60 years ago with the development of electronic distance measuring (EDM) instruments. These devices measure lengths by indirectly determining the number of full and partial waves of transmitted electromagnetic energy required in traveling between the two ends of a line. In practice, the energy is transmitted from one end of the line to the other and returned to the starting point; thus, it travels the double path distance. Multiplying the total number of cycles by its wavelength and dividing by 2, yields the unknown distance.

Electronic distance measurement is based on the rate and manner that electromagnetic energy propagates through the atmosphere. The rate of propagation can be expressed with the following equation

$$V = \lambda f$$

where V is the velocity of electromagnetic energy, in meters per second; f the modulated frequency of the energy, in hertz; 2 and λ the wavelength, in meters. The velocity of electromagnetic energy in a vacuum is 299,792,458 m/sec. Its speed is slowed somewhat in the atmosphere according to the following equation

$$V = c/n$$

where c is the velocity of electromagnetic energy in a vacuum, and n the atmospheric index of refraction. The value of n varies from about 1.0001 to 1.0005, depending on pressure and temperature, but is approximately equal to 1.0003. Thus, accurate electronic distance measurement requires that atmospheric pressure and temperature be measured so that the appropriate value of n is known.

University of Anbar
College of Engineering
Civil Engineering Department

LECTURE NOTE
ENGINEERING SURVEYING

Chapter Five

By

Dr. Maher Shakir Dr. Hameed Aswad

CHAPTER FIVE

Angle Measurement

Determining the locations of points and orientations of lines frequently depends on the observation of angles and directions. In surveying, directions are given by *azimuths* and *bearings.*

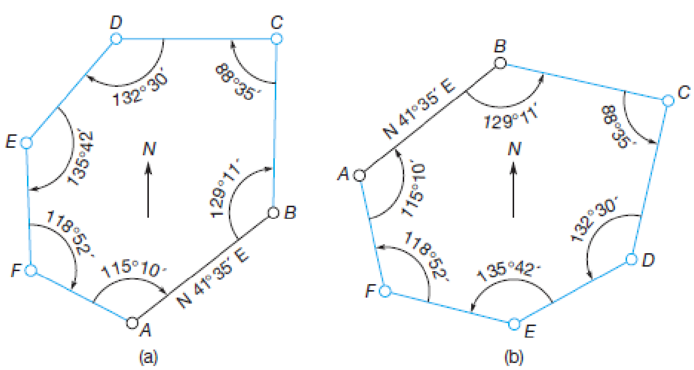
- An **angle** is defined as the difference in direction between two convergent lines.

- **A horizontal angle** is formed by the directions to two objects in a horizontal plane. Horizontal angles are the basic observations needed for determining bearings and azimuths.

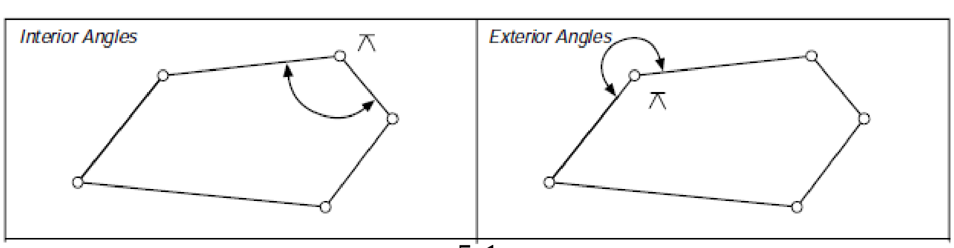
- **A vertical angle** is formed by two intersecting lines in a vertical plane, one of these lines horizontal. Vertical angles are used in trigonometric leveling, stadia, and for reducing slope distances to horizontal.

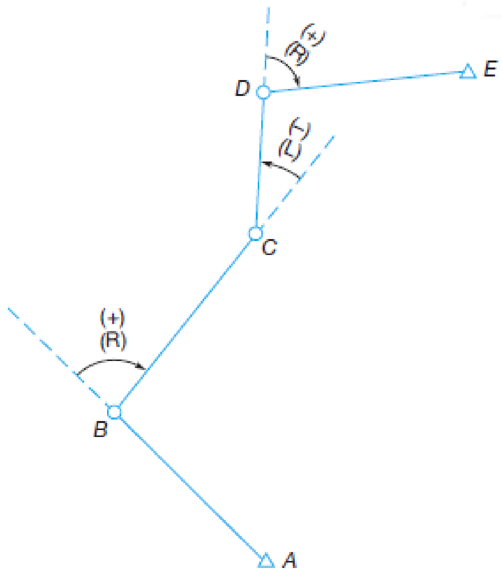
Types of Measured Angles

- ***Interior angles*** are observed on the inside of a closed polygon. Normally the angle at each apex within the polygon is measured. Then, a check can be made on their values because the sum of all interior angles in any polygon must equal (n - 2) 180° where n is the number of angles. Polygons are commonly used for boundary surveys and many other types of work. Surveyors (geomatics engineers) normally refer to them as closed traverses.
- ***Exterior angles***, located outside a closed polygon, are explements of interior angles. The advantage to be gained by observing them is their use as another check, since the sum of the interior and exterior angles at any station must total 360°.
- ***Deflection angles***, right or left, are measured from an extension of the preceding course and the ahead line. It must be noted when the deflection is right (R) or left (L).



Closed polygon.
(a) Clockwise interior angles (angles to the right).
(b) Counterclock-wise interior angles (angles to the left).



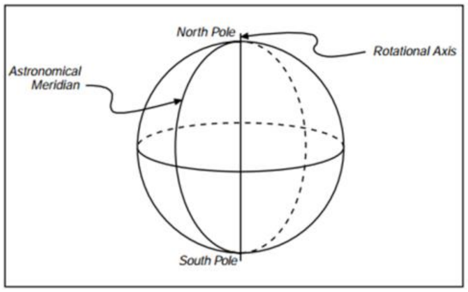


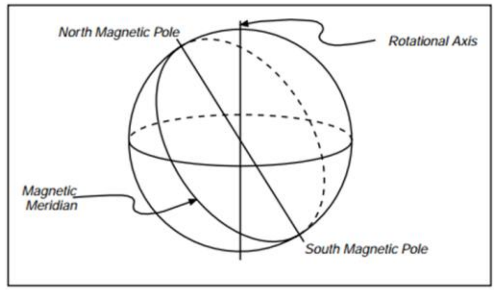
Deflection angles.

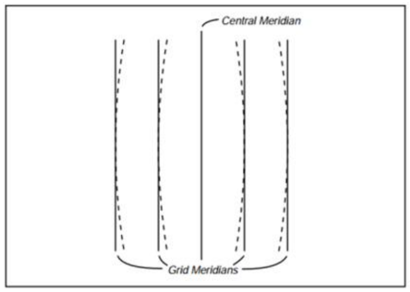
Direction of a Line

The direction of a line is defined by the horizontal angle between the line and an arbitrarily chosen reference line called a meridian. Different meridians are used for specifying directions including (a) geodetic (also often called true), (b) astronomic, (c) magnetic, (d) grid, (e) record, and (f) assumed.

- The ***geodetic meridian*** is the north-south reference line that passes through a mean position of the Earth's geographic poles.
- The ***astronomic meridian*** is the north-south reference line that passes through the instantaneous position of the Earth's geographic poles. Astronomic meridians consists in making observations on the celestial objects. Geodetic and astronomic meridians are very nearly the same, and the former can be computed from the latter by making small corrections.
- A ***magnetic meridian*** is defined by a freely suspended magnetic needle that is only influenced by the Earth's magnetic field.
- Surveys based on a state or other plane coordinate system employ a ***grid meridian*** for reference. Grid north is the direction of geodetic north for a selected central meridian and held parallel to it over the entire area covered by a plane coordinate system.
- In boundary surveys, the term ***record meridian*** refers to directional references quoted in the recorded documents from a previous survey of a particular parcel of land.
- An ***assumed meridian*** can be established by merely assigning any arbitrary direction—for example, taking a certain street line to be north. The directions of all other lines are then found in relation to it.

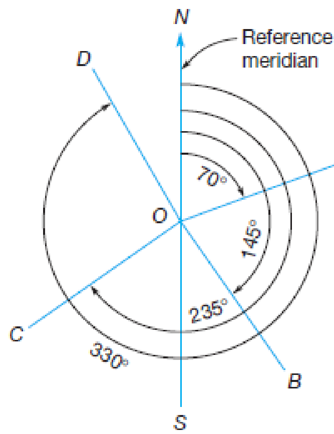






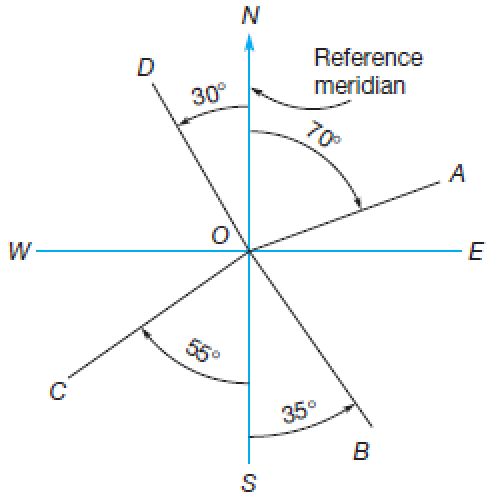
Bearings and Azimuths

Azimuths are horizontal angles observed clockwise from any reference meridian. In plane surveying, azimuths are generally observed from north, but astronomers and the military have used south as the reference direction. Examples of azimuths observed from north are shown in below figure. As illustrated, they can range from 0° to 360° in value. Thus the azimuth of OA is 70°; of OB, 145°; of OC, 235°; and of OD, 330°. Azimuths may be geodetic, astronomic, magnetic, grid, record, or assumed, depending on the reference meridian used.



Azimuths

Bearings are another system for designating directions of lines. The bearing of a line is defined as the acute horizontal angle between a reference meridian and the line. The angle is observed from either the north or south toward the east or west, to give a reading smaller than 90°.The letter N or S preceding the angle, and E or W following it shows the proper quadrant. Thus, a properly expressed bearing includes quadrant letters and an angular value. An example is N80°E. In below figure:



Bearing angles

Comparison of Azimuths and Bearings

Azimuths	**Bearings**
Vary from 0 to 360°	Vary from 0 to 90°
Require only a numerical value	Require two letters and a numerical value
May be geodetic, astronomic, magnetic, grid, assumed, forward or back	Same as azimuths
Are measured clockwise only	Are measured clockwise and counterclockwise
Are measured either from north only, or from south only on a particular survey	Are measured from north and south

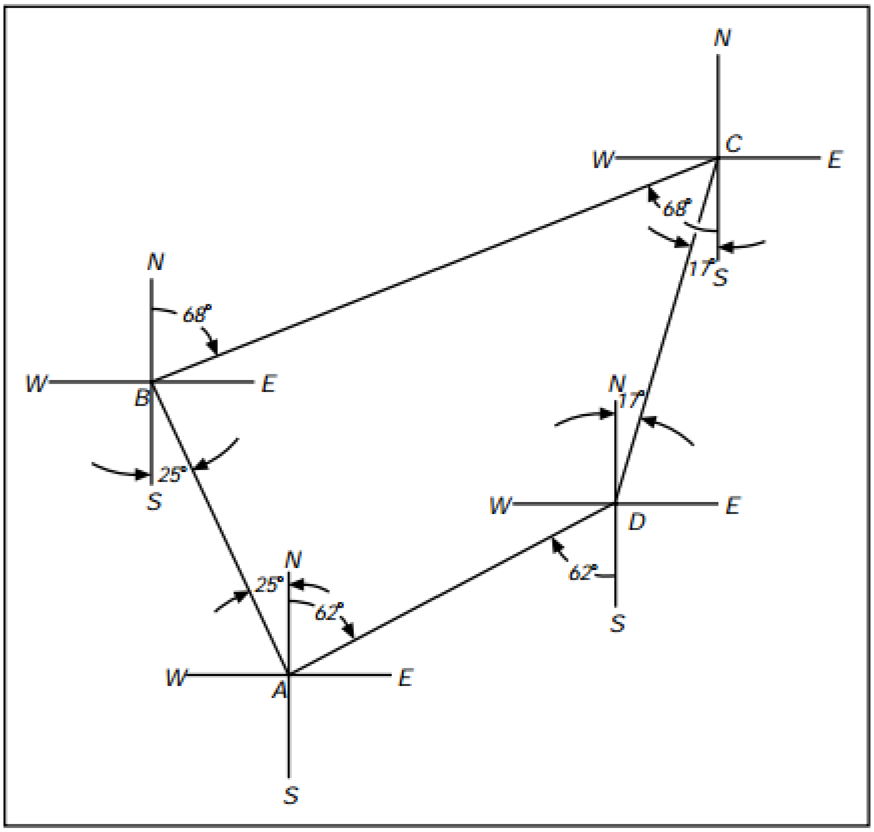
Quadrant	**Formulas for computing bearing angles from azimuths**
I (NE)	Bearing = Azimuth
II (SE)	Bearing = 180° − Azimuth
III (SW)	Bearing = Azimuth − 180°
IV (NW)	Bearing = 360° − Azimuth

Example:
The first course of a boundary survey is written as N37°13.W. What is its equivalent azimuth?

Solution
Since the bearing is in the northwest quadrant, the azimuth is

360° - 37°13. = 322°47.

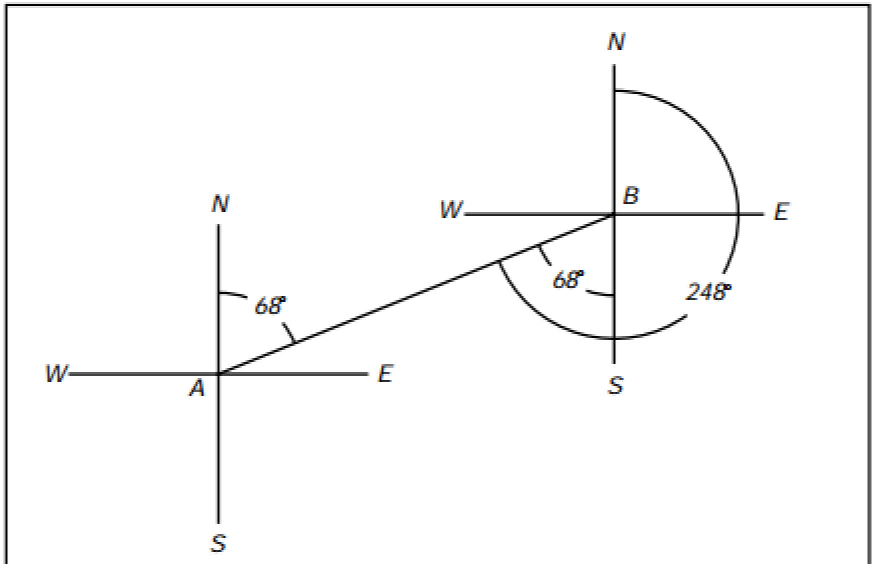
For the figure below, calculate the bearings for each line.



Line	Bearing
A – B	
B – C	
C – D	
D – A	

Back Azimuths and Back Bearings

The back azimuth or back bearing of a line is the azimuth or bearing of a line running in the reverse direction. The azimuth or bearing of a line in the direction in which a survey is progressing is called the forward azimuth or forward bearing. The azimuth or bearing of the line in the direction opposite to that of progress is called the back azimuth or back bearing. The back azimuth can be obtained by adding 180° if the azimuth is less than 180° or by subtracting 180° if the azimuth is greater than 180°. The back bearing can be obtained from the forward bearing by changing the first letter from N to S or from S to N and the second letter from E to W or from W to E.

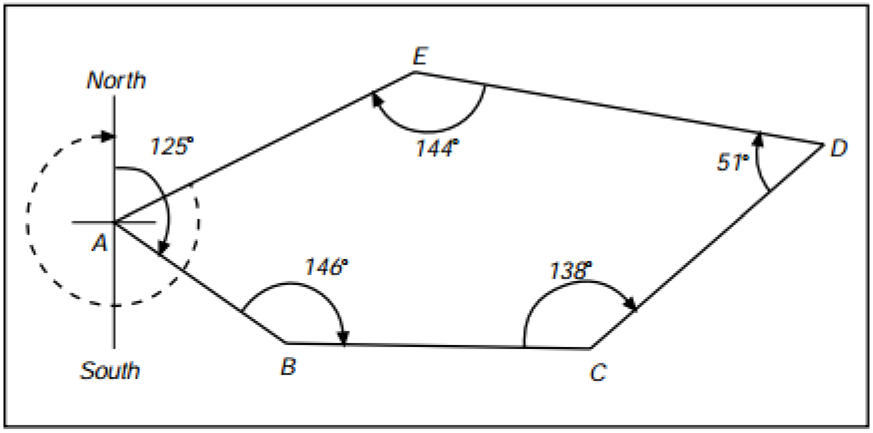


Bearing or Azimuth and Back Bearing or Back Azimuth

The bearing of the line A-B is N 68° E.
The azimuth of the line A-B is 68°.

The bearing of the line B-A is S 68° W.
The azimuth of the line B-A is 248°.

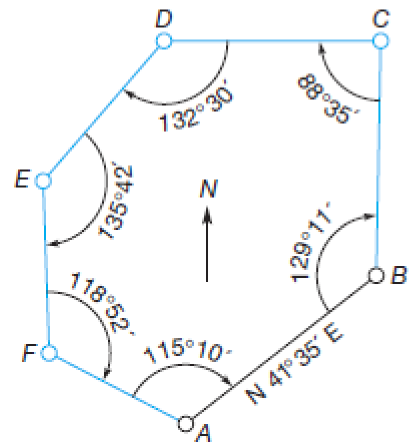
Example:



Solution

Line	Bearing	Azimuth
A – B	S 55° E	125°
B – C	S 89° E	91°
C – D	N 49° E	49°
D – E	N 80° W	280°
E – A	S 64° W	244°

Example 2: determine azimuth for each line?



$41°35' = AB$	$211°51' = DE$
$+180°00'$	$-180°00'$
$221°35' = BA$	$31°51' = ED$
$+129°11'$	$+135°42'$
$350°46' = BC$	$167°33' = EF$
$-180°00'$	$+180°00'$
$170°46' = CB$	$347°33' = FE$
$+88°35'$	$+118°52'$
$259°21' = CD$	$466°25' - {}^*360° = 106°25' = FA$
$-180°00'$	$-180°00'$
$79°21' = DC$	$286°25' = AF$
$+132°30'$	$+115°10'$
$211°51' = DE$	$401°35' - {}^*360° = 41°35' = AB$ ✔

Example 3:

The angles at the stations of a closed traverse *ABCDEFA* were observed as given below:

Traverse station	Interior angle
A	120°35′00″
B	89°24′20″
C	131°01′00″
D	128°02′20″
E	94°54′40″
F	155°59′20″

calculate the azimuth of the traverse lines in the following systems if azimuth of the line *AB* is 42°:

W.C.B. of AB = 42° (given)
W.C.B. of BA = 180° + 42° = 222°00′00″
W.C.B. of BC = W.C.B. of BA − ∠B
= 222°00′00″ − 89°24′20″ = 132°35′40″
W.C.B. of CB = 180° + 132°35′40″ = 312°35′40″
W.C.B. of CD = W.C.B. of CB − ∠C
= 312°25′40″ − 131°01′40″ = 181°34′00″
W.C.B. of DC = 180° + 181°34′00″ = 361°34′00″ − 360° = 1°34′00″
W.C.B. of DE = W.C.B. of DC − ∠D
= 361°34′00″ − 128°03′00″ = 233°31′00″
W.C.B. of ED = 180° + 233°31′00″ = 413°31′00″ − 360° = 53°31′00″
W.C.B. of EF = W.C.B. of ED − ∠E
= 413°31′00″ − 94°55′20″ = 318°35′40″
W.C.B. of FE = 180° + 318°35′40″ = 498°35′40″ − 360° = 138°35′40″
W.C.B. of FA = W.C.B. of FE − ∠F
= 498°35′40″ − 156°00′00″ = 342°35′40″
W.C.B. of AF = 180° + 342°35′40″ = 522°35′40″ − 360° = 162°35′40″
W.C.B. of AB = W.C.B. of AF − ∠A
= 162°35′40″ − 120°35′40″ = 42°00′00″. (*Check*)

Magnetic declination is the horizontal angle observed from the geodetic meridian to the magnetic meridian. Navigators call this angle *variation* of the compass; the armed forces use the term *deviation.* An east declination exists if the magnetic meridian is east of geodetic north; a west declination occurs if it is west of geodetic north. East declinations are considered positive and west declinations negative.
The relationship between geodetic north, magnetic north, and magnetic declination is given by the expression

Geodetic azimuth = magnetic azimuth + magnetic declination

Methods of Measuring Angles

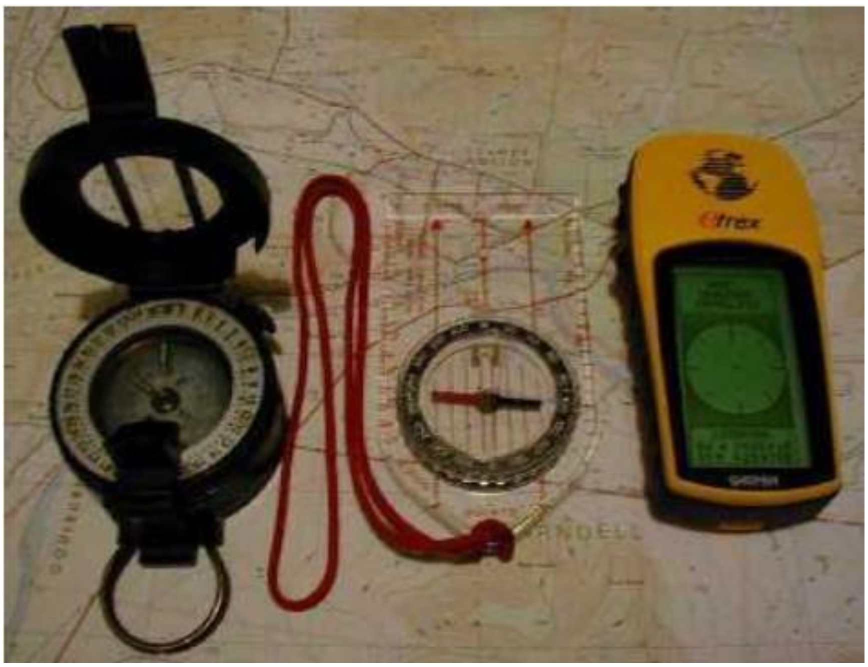
Angles normally are measured with a theodolite or total station system, but also can be determined by means of a tape or a compass.

*Tapes***:** Here the angle is not directly measured rather calculated from the measurement of distance. The accuracy depends on the accuracy of measurement of the distance. For acute angles on level ground, the error needs not exceed 05’to 10’. For obtuse angle, the corresponding acute angle should be determined. This method is generally slow and is used in absence of direct measuring instruments and as check.

Theodolite: are used for measuring both vertical and horizontal angles with an accuracy of 10’’for horizontal and 1’ for vertical angles.

Total station: incorporate both angle and distance measurement. The accuracy of measuring angles is 20’’to 1’’.

Compass: In compass survey, the direction of the survey line is measured by the use of a magnetic compass while the lengths are by chaining or taping. Where the area to be surveyed is comparatively large, the compass survey is preferred. However, where the compass survey is used, care must be taken to make sure that magnetic disturbances are not present. The two major primary types of survey compass are: the prismatic compass and surveyors compass



Compass surveys are mainly used for the rapid filling of the detail in larger surveys and for explanatory works. It does not provide a very accurate determination of the bearing of a line as the compass needle aligns itself to the earth’s magnetic field which does not provide a constant reference point.

1. *Prismatic compass:* This is an instrument used for the measurement of magnetic bearings. It is small and portable usually carried on the hand. This Prismatic Compass is one of the two main kinds of magnetic compasses included in the collection for the purpose of measuring magnetic bearings, with the other being the Surveyor's Compass. The main difference between the two instruments is that the surveyor's compass is usually larger and more accurate instrument, and is generally used on a stand or tripod.
2. *Surveyor’s Compass:* Similar to the prismatic compass but with few modifications, the surveyors compass is an old form of compass used by surveyors. It is used to determine the magnetic bearing of a given line and is usually used in connection with the chain or compass survey.

University of Anbar

College of Engineering

Civil Engineering Department

LECTURE NOTE
ENGINEERING SURVEYING

Chapter Six
Traversing

By

Dr. Maher Shakir **Dr. Hameed Aswad**

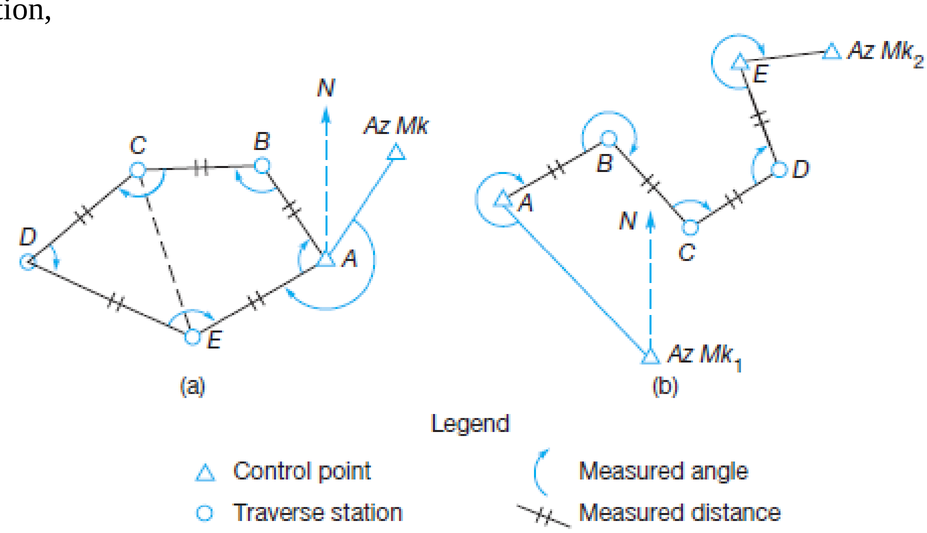
CHAPTER SIX

Traversing

Definition of a traverse

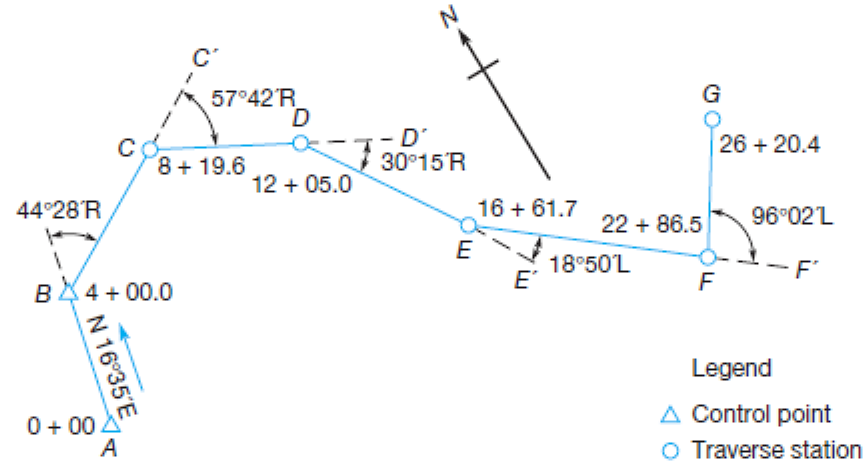
A traverse is a series of consecutive lines whose ends have been marked in the field and whose lengths and directions have been determined from observations. In traditional surveying by ground methods, traversing, the act of marking the lines, that is, establishing traverse stations and making the necessary observations, is one of the most basic and widely practiced means of determining the relative locations of points.

- There are two kinds of traverses: closed and open. Two categories of closed traverses exist: polygon and link.
- In the polygon traverse, the lines return to the starting point, thus forming a closed figure that is both geometrically and mathematically closed. Link traverses finish upon another station that should have a positional accuracy equal to or greater than that of the starting point. The link type (geometrically open, mathematically closed) must have a closing reference direction,



Examples of closed traverses.

- An open traverse (geometrically and mathematically open) consists of a series of lines that are connected but do not return to the starting point or close upon a point of equal or greater order accuracy.



Open traverse.

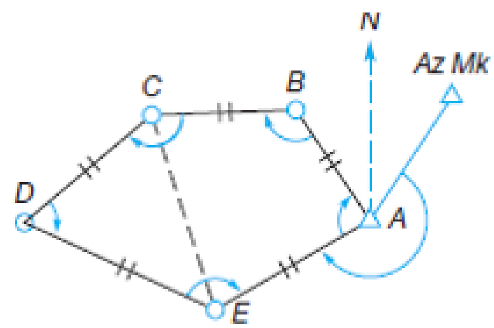
Observation of Traverse Angles or Directions

The methods used in observing angles or directions of traverse lines vary and include (1) interior angles, (2) angles to the right, (3) deflection angles, and (4) azimuths. These are described in the following subsections.

1. Traversing by Interior Angles

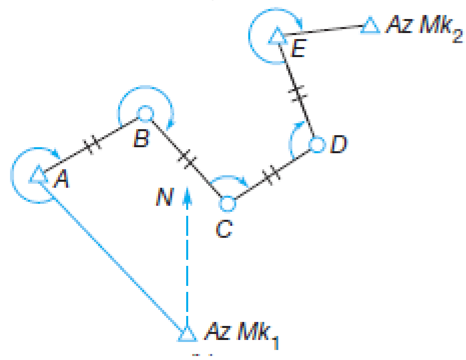
Although interior angles could be observed either clockwise or counterclockwise, to reduce mistakes in reading, recording, and computing, they should always be turned clockwise from the backsight station to the foresight station. For example, angle EAB of below figure was observed at station A, with the backsight on station E and the foresight at station B.

- Interior angles may be improved by averaging equal numbers of direct and reversed readings.



2. Traversing by Angles to the Right

Depending on the direction of the traversing, angles to the right may be interior or exterior angles in a polygon traverse. If the direction of traversing is counter clockwise around the figure, then clockwise interior angles will be observed. However, if the direction of traversing is clockwise, then exterior angles will be observed. Data collectors generally follow this convention when traversing. Thus, in below figure, for example, the direction from *A* to *B*, *B* to *C*, *C* to *D*, etc., is forward. By averaging equal numbers of direct and reversed readings, observed angles to the right can also be checked and their accuracy improved.

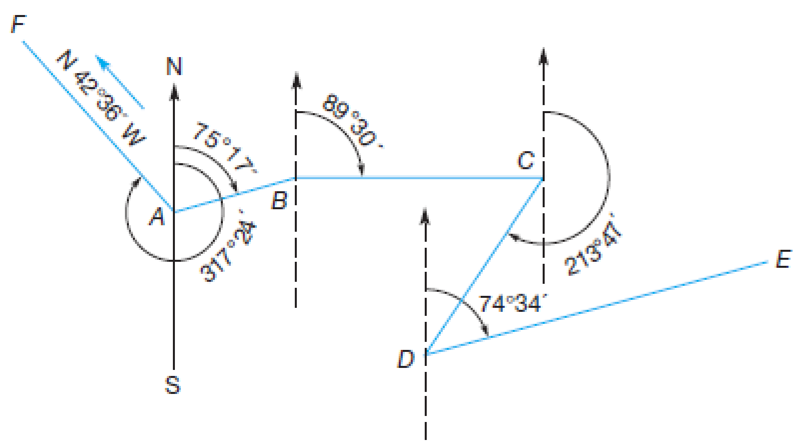


3. Traversing by Deflection Angles

A deflection angle is not complete without a designation R or L, and, of course, it cannot exceed 180°. Each angle should be doubled or quadrupled, and an average value determined. The angles should be observed an equal number of times in face left and face right to reduce instrumental errors. Deflection angles can be obtained by subtracting 180° from angles to the right. Positive values so obtained denote right deflection angles; negative ones are left.

4. Traversing by Azimuths

As shown in the below figure, azimuths are observed clockwise from the north end of the meridian through the angle points. The instrument is oriented at each setup by sighting on the previous station with either the back azimuth on the circle (if angles to the right are turned) or the azimuth (if deflection angles are turned).Then the forward station is sighted. The resulting reading on the horizontal circle will be the forward line's azimuth.



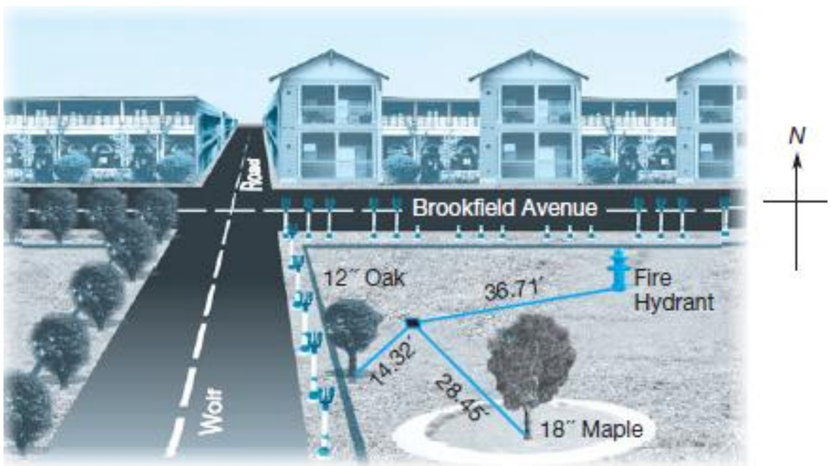
Azimuth traverse.

Observation of Traverse Lengths

The length of each traverse line (also called a course) must be observed, and this is usually done by the simplest and most economical method such as tape, total station etc. In closed traverses, each course is observed and recorded as a separate distance. On long link traverses for highways and railroads, distances are carried along continuously from the starting point using stationing. for example, beginning with station 0+00 at point A, 100-m stations (1+00, 2+00, 3+00) are marked until hub B at station 4+00 is reached., and the end 8 + 19.60. The length of a line in a stationed link traverse is the difference between stationing at its end points; thus, the length of line BC is 819.60 - 400.00 = 419.60 m.

Referencing Traverse Stations

Traverse stations often must be found and reoccupied months or even years after they are established. Also they may be destroyed through construction or other activity. Therefore, it is important that they be referenced by creating observational ties to them so that they can be relocated if obscured or re-established if destroyed



Angle Misclosure

The angular misclosure for an interior-angle traverse is the difference between the sum of the observed angles and the geometrically correct total for the polygon. The sum, Σ, of the interior angles of a closed polygon should be

$$\Sigma = (n - 2)\ 180°$$

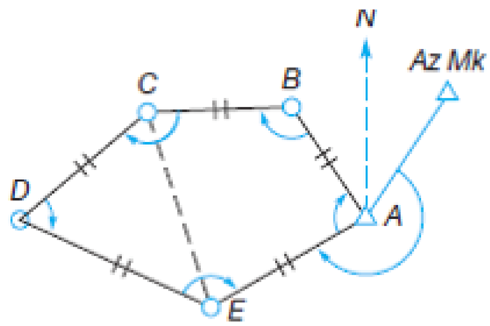
Where n is the number of sides, or angles, in the polygon. The sum of the angles in a triangle is 180°; in a rectangle, 360°; and in a pentagon, 540°.Thus, each side added to the three required for a triangle increases the sum of the angles by 180°. If the direction about a traverse is clockwise when observing angles to the right, exterior angles will be observed. In this case, the sum of the exterior angles will be

$$\Sigma = (n + 2)\ 180°$$

Figure shows a five-sided figure in which, if the sum of the observed interior angles equals 540°00'05" the angular misclosure is 5". Misclosures result from the accumulation of random errors in the angle observations. Permissible misclosure can be computed by the formula

$$c = K\sqrt{n}$$

Where n is the number of angles, and K a constant that depends on the level of accuracy specified for the survey.



Sources of Error in Traversing

Some sources of error in running a traverse are:

1. Poor selection of stations, resulting in bad sighting conditions caused by (a) alternate sun and shadow, (b) visibility of only the rod's top, (c) line of sight passing too close to the ground, (d) lines that are too short, and (e) sighting into the sun.
2. Errors in observations of angles and distances.
3. Failure to observe angles an equal number of times direct and reversed.

Mistakes in Traversing

Some mistakes in traversing are:

1. Occupying or sighting on the wrong station.
2. Incorrect orientation.
3. Confusing angles to the right and left.
4. Mistakes in note taking.
5. Misidentification of the sighted station.

Balancing Angles

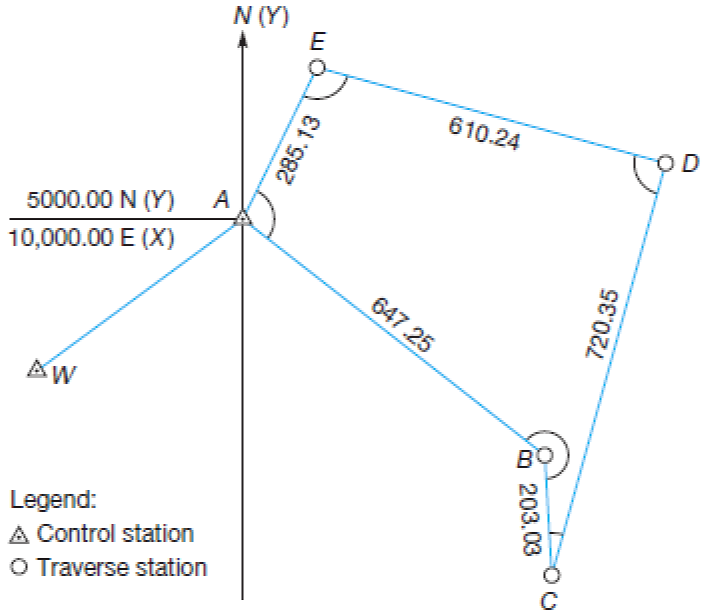
In elementary methods of traverse adjustment, the first step is to balance (adjust) the angles to the proper geometric total. For closed traverses, angle balancing is done readily since the total error is known, although its exact distribution is not. Angles of a closed traverse can be adjusted to the correct geometric total by applying one of two methods:

1. Applying an average correction to each angle where observing conditions were approximately the same at all stations. The correction for each angle is found by dividing the total angular misclosure by the number of angles.
2. Making larger corrections to angles where poor observing conditions were present.

Of these two methods, the first is almost always applied.

Example

For the traverse of the figure, the observed interior angles are given in the table. Compute the adjusted angles using methods 1 and 2.



Point (1)	Measured Interior Angle (2)	Method 1: Multiples of Average Correction (3)	Correction Rounded To 1″ (3)	Successive Differences (5)	Adjusted Angle (6)
A	100°45′37″	2.2″	2″	2″	100°45′35″
B	231°23′43″	4.4″	4″	2″	231°23′41″
C	17°12′59″	6.6″	7″	3″	17°12′56″
D	89°03′28″	8.8″	9″	2″	89°03′26″
E	101°34′24″	11.0″	11″	2″	101°34′22″
	Σ = 540°00′11″			Σ = 11″	Σ = 540°00′00″

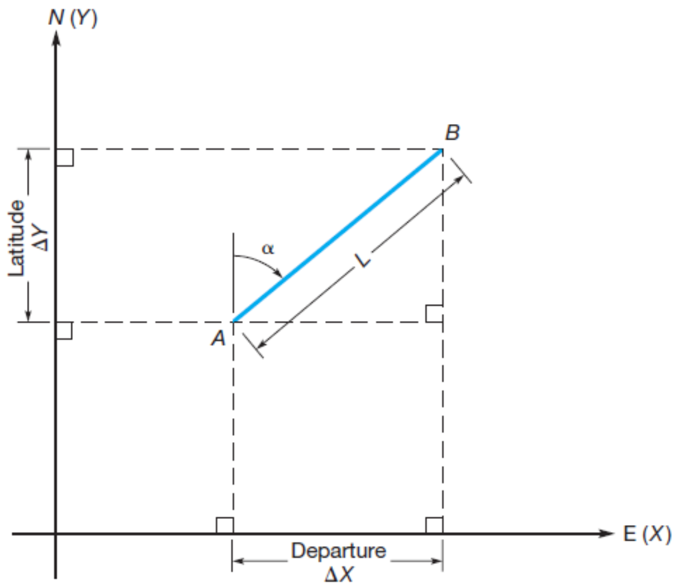
Point (1)	Measured Interior Angle (2)	Method 2: Adjustment (7)	Adjusted Angle (8)
A	100°45′37″	2″	100°45′35″
B	231°23′43″	3″	231°23′40″
C	17°12′59″	3″	17°12′56″
D	89°03′28″	1″	89°03′27″
E	101°34′24″	2″	101°34′22″
	Σ = 540°00′11″	Σ = 11″	Σ = 540°00′00″

Departures and Latitudes

After balancing the angles and calculating preliminary azimuths (or bearings), traverse closure is checked by computing the *departure* and *latitude* of each line. As illustrated in the figure, the departure of a course is its orthographic projection on the east-west axis of the survey and is equal to the length of the course multiplied by the sine of its azimuth (or bearing) angle. Departures are sometimes called eastings or westings. The latitude of a course is its orthographic projection on the north-south axis of the survey, and is equal to the course length multiplied by the cosine of its azimuth (or bearing) angle. Latitude is also called northing or southing.

In equation form, the departure and latitude of a line are:

$$Departure = L \times \sin\alpha$$
$$Latitude = L \times \cos\alpha$$



	Quadrant			
	N-E	S-E	S-W	N-W
Departure	+	+	–	–
Latitude	+	–	–	+

- In traverse calculations, east departures and north latitudes are considered plus; west departures and south latitudes, minus.

Traverse Linear Misclosure and Relative Precision

- All angles and distances were measured perfectly, the algebraic sum of the departures of all courses in the traverse should equal zero. Likewise, the algebraic sum of all latitudes should equal zero.

- Because the observations are not perfect and errors exist in the angles and distances, the conditions just stated rarely occur. The amounts by which they fail to be met are termed *departure misclosure* and *latitude misclosure*. Their values are computed by algebraically summing the departures and latitudes, and comparing the totals to the required conditions.

- The linear misclosure of the traverse is calculated from the following formula:

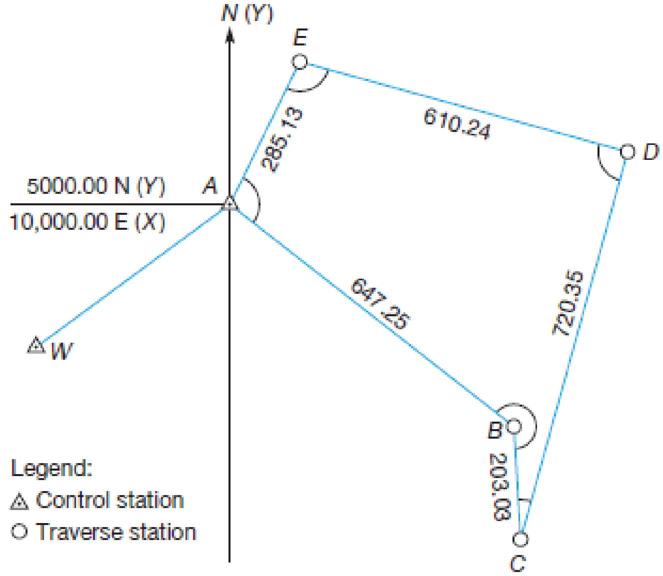
$$\text{linear misclosure} = \sqrt{(\text{departure misclosure})^2 + (\text{latitude misclosure})^2}$$

- The relative precision of a traverse is expressed by a fraction that has the linear misclosure as its numerator and the traverse perimeter or total length as its denominator, or

$$\text{relative precision} = \frac{\text{linear misclosure}}{\text{traverse length}}$$

Example

Based on the preliminary azimuths from table and lengths shown in the below, calculate the departures and latitudes, linear misclosure, and relative precision of the traverse.



Station	Preliminary Azimuths	Length	Departure	Latitude
A	126°55′17″	647.25	517.451	−388.815
B	178°18′58″	203.03	5.966	−202.942
C	15°31′54″	720.35	192.889	694.045
D	284°35′20″	610.24	−590.565	153.708
E	206°09′42″	285.13	−125.715	−255.919
A		Σ = 2466.00	Σ = 0.026	Σ = 0.077

$$\text{linear misclosure} = \sqrt{(0.026)^2 + (0.077)^2} = 0.081$$

The relative precision for this traverse is

$$\text{relative precision} = \frac{0.081}{2466.00} = \frac{1}{30{,}000}$$

TRAVERSE ADJUSTMENT

For any closed traverse, the linear misclosure must be adjusted (or distributed) throughout the traverse to “close” or “balance” the figure. This is true even though the misclosure is negligible in plotting the traverse at map scale. There are several methods available for traverse adjustment, but the one most commonly used is the *compass rule* (Bowditch method). As noted earlier, adjustment by least squares is a more advanced technique that can also be used.

The compass, or Bowditch, rule adjusts the departures and latitudes of traverse courses in proportion to their lengths. Corrections by this method are made according to the following rules:

$$\text{correction in departure for } AB = -\frac{(\text{total departure misclosure})}{\text{traverse perimeter}} \text{length of } AB$$

$$\text{correction in latitude for } AB = -\frac{(\text{total latitude misclosure})}{\text{traverse perimeter}} \text{length of } AB$$

Example

Using the preliminary azimuths and lengths from previous table, compute departures and latitudes, linear misclosure, and relative precision. Balance the departures and latitudes using the compass rule.

Solution:

The correction in departure for AB is

$$-\left(\frac{0.026}{2466}\right)647.25 = -0.007$$

And by Equation (10.6) the correction for the latitude of AB

$$-\left(\frac{0.077}{2466}\right)647.25 = -0.020$$

Station	Preliminary Azimuths	Length (ft)	Unadjusted Departure	Unadjusted Latitude	Balanced Departure	Balanced Latitude	Coordinates* X (ft) (easting)	Coordinates* Y (ft) (northing)
A			(−0.007)	(−0.020)			10,000.00	5000.00
	126°55′17″	647.25	517.451	−388.815	517.444	−388.835		
B			(−0.002)	(−0.006)			10,517.44	4611.16
	178°18′58″	203.03	5.966	−202.942	5.964	−202.948		
C			(−0.008)	(−0.023)			10,523.41	4408.22
	15°31′54″	720.35	192.889	694.045	192.881	694.022		
D			(−0.006)	(−0.019)			10,716.29	5102.24
	284°35′20″	610.24	−590.565	153.708	−590.571	153.689		
E			(−0.003)	(−0.009)			10,125.72	5255.93
	206°09′42″	285.13	−125.715	−255.919	−125.718	−255.928		
A							10,000.00✓	5000.00✓
		Σ = 2466.00	Σ = 0.026	Σ = 0.077	Σ = 0.000	Σ = 0.000		

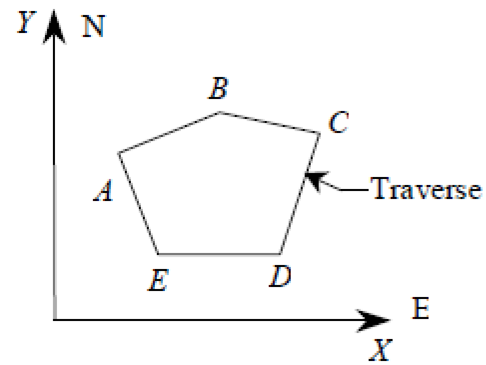
$$\text{Linear precision} = \sqrt{(0.026)^2 + (-0.077)^2} = 0.081 \text{ ft}$$

$$\text{Relative precision} = \frac{0.081}{2466} = \frac{1}{30{,}000}$$

*Coordinates are rounded to same significance as observed lengths.

COORDINATES

Normally, plane rectangular coordinate system (Cartesian plane) having x-axis in east-west direction and y-axis in north-south direction, is used to define the location of the traverse stations. The y-axis is taken as the reference axis and it can be (a) true north, (b) magnetic north, (c) National Grid north, or (d) a chosen arbitrary direction.



Usually, the origin of the coordinate system is so placed that the entire traverse falls in the first quadrant of the coordinate system and all the traverse stations have positive coordinates as shown in figure.

Given the X and Y coordinates of any starting point A, the X coordinate of the next point B is obtained by adding the adjusted departure of course AB to X_A. Likewise, the Y coordinate of B is the adjusted latitude of AB added to Y_A. In equation form this is

$$X_B = X_A + \text{departure } AB$$

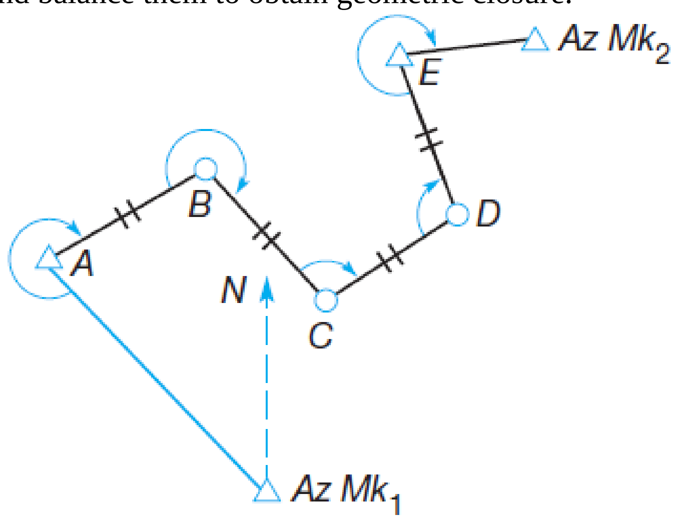
$$Y_B = Y_A + \text{latitude} AB$$

Alternative Methods for Making Traverse Computations

A. Balancing Angles by Adjusting Azimuths or Bearings

Example:

Table lists observed angles to the right for the traverse of figure. The azimuths of lines A-AzMk$_1$ and E-AzMk$_2$ have known values of 139°05' 45" and 86°20'47" respectively. Compute unadjusted azimuths and balance them to obtain geometric closure.



Station (1)	Measured Angle* (2)	Unadjusted Azimuth (3)	Azimuth Correction (4)	Preliminary Azimuth (5)
Az MK$_1$				
		319°05′45″		319°05′45″
A	283°50′10″			
		62°55′55″	−2″	62°55′53″
B	256°17′18″			
		139°13′13″	−4″	139°13′09″
C	98°12′36″			
		57°25′49″	−6″	57°25′43″
D	103°30′34″			
		340°56′23″	−8″	340°56′15″
E	285°24′34″			
		86°20′57″	−10″	86°20′47″
Az Mk$_2$				

86°20′57″
−86°20′47″
misclosure = 0°00′10″
correction per angle = −10″/5 = −2″

*Observed angles are angles to the right.

B. Balancing Departures and Latitudes by Adjusting Coordinates

Example

The table lists the preliminary azimuths and observed lengths for the traverse of previous example. The known coordinates of stations *A* and *E* are X_A = 12,765.48, Y_A = 43,280.21, X_E = 14,797.12, and Y_E = 44,384.51. Adjust this traverse for departure and latitude misclosures by making corrections to preliminary coordinates.

Solution:

Station (1)	Length (ft) (2)	Preliminary Azimuth (3)	Departure (4)	Latitude (5)	Preliminary Coordinates (ft) X (6)	Y (7)	Corrections (ft) X (8)	Y (9)	Adjusted Coordinates* X (ft) (10)	Y (ft) (11)
A					12,765.48	43,280.21			12,765.48	43,280.21
	1045.50	62°55′53″	930.978	475.762			−0.048	0.006		
B					13,696.458	43,755.972	(−0.048)	(0.006)	13,696.41	43,755.98
	1007.38	139°13′09″	657.988	−762.802			−0.046	0.006		
C					14,354.446	42,993.170	(−0.094)	(0.012)	14,354.35	42,993.18
	897.81	57°25′43″	756.604	483.336			−0.041	0.006		
D					15,111.050	43,476.506	(−0.135)	(0.018)	15,110.92	43,476.52
	960.66	340°56′15″	−313.751	907.980			−0.044	0.006		
E					14,797.299	44,384.486	(−0.179)	(0.024)	14,797.12✔	44,384.51✔
	Σ = 3911.35				−14,797.12	−44,384.51				
				Misclosures	+0.179	−0.024				

$$\text{Linear precision} = \sqrt{(0.179)^2 + (-0.024)^2} = 0.181 \text{ ft}$$

$$\text{Relative precision} = \frac{0.181}{3911} = \frac{1}{21{,}000}$$

*Adjusted coordinates are rounded to same significance as observed lengths.

Inversing

If the departure and latitude of a line AB are known, its length and azimuth or bearing are readily obtained from the following relationships:

$$\tan azimuth\ (or\ bearing)\ AB = \frac{departure\ AB}{latitude\ AB}$$

$$length\ AB = \frac{departure\ AB}{\sin azimuth\ (or\ bearing) AB} = \frac{latitude\ AB}{\cos azimuth\ (or\ bearing) AB}$$

$$length\ AB = \sqrt{(departure\ AB)^2 + (latitude\ AB)^2}$$

It can be written to express departures and latitudes in terms of coordinate differences as follows:

$$departure\ AB = X_B - X_A = \Delta X$$

$$latitude\ AB = Y_B - Y_A = \Delta Y$$

$$length\ AB = \frac{\Delta X}{\sin azimuth\ (or\ bearing) AB} = \frac{\Delta Y}{\cos azimuth\ (or\ bearing) AB}$$

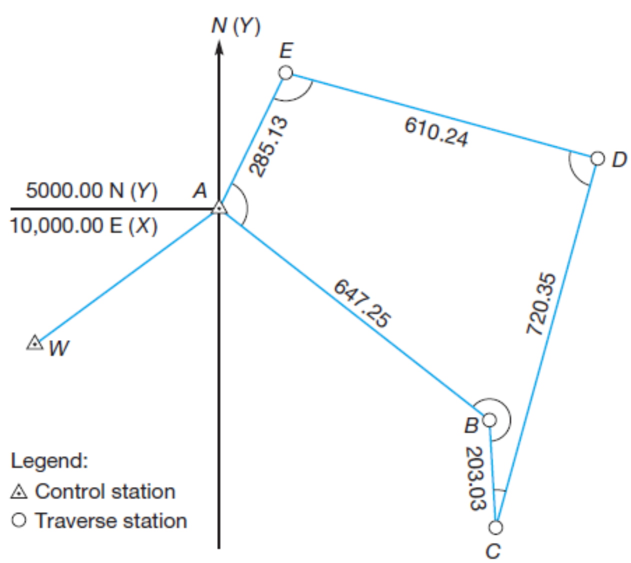
$$length\ AB = \sqrt{(\Delta X)^2 + (\Delta Y)^2}$$

Computing Final Adjusted Traverse Lengths and Directions

In traverse adjustments, corrections are applied to the computed departures and latitudes to obtain adjusted values. These in turn are used to calculate X and Y coordinates of the traverse stations. By changing departures and latitudes of lines in the adjustment process, their lengths and azimuths (or bearings) also change.

Example

Calculate the final adjusted lengths and azimuths of the traverse as shown in figure from the adjusted departures and latitudes listed in the table.



	Balanced		Balanced	
Line	Departure	Latitude	Length (ft)	Azimuth
AB	517.444	−388.835	647.26	126°55′23″
BC	5.964	−202.948	203.04	178°19′00″
CD	192.881	694.022	720.33	15°31′54″
DE	−590.571	153.689	610.24	284°35′13″
EA	−125.718	−255.928	285.14	206°09′41″

$$\tan \text{azimuth}_{AB} = \frac{517.444}{-388.835} = -1.330755;$$

$$\text{azimuth}_{AB} = -53°04'37'' + 180° = 126°55'23''$$

$$\text{length}_{AB} = \sqrt{(517.444)^2 + (-388.835)^2} = 647.26 \text{ ft}$$

Angle	Foresight Azimuth	Backsight Azimuth	Adjusted Angle	Difference
A (EAB)	*AB* = 126°55′23″	*AE* = 26°09′41″	100°45′42″	7″
B (ABC)	*BC* = (178°19′00″ + 360°)	*BA* = 306°55′23″	231°23′37″	−4″
C (BCD)	*CD* = (15°31′54″ + 360°)	*CB* = (178°19′00″ + 180°)	17°12′54″	−2″
D (CDE)	*DE* = 284°35′13″	*DC* = (15°31′54″ + 180°)	89°03′19″	−7″
E (DEA)	*EA* = 206°09′41″	*ED* = (284°35′13″ − 180°)	101°34′28″	6″
			$\sum$ = 540°00′00″	$\sum$ = 0″

Example:
Using coordinates, calculate adjusted lengths and azimuths for the traverse of previous example.

Station	Adjusted Coordinates	
	X	Y
A	12,765.48	43,280.21
B	13,696.41	43,755.98
C	14,354.35	42,993.18
D	15,110.92	43,476.52
E	14,797.12	44,384.51

Solution:

$$X_B - X_A = 13{,}696.41 - 12{,}765.48 = 930.93 = \Delta X$$

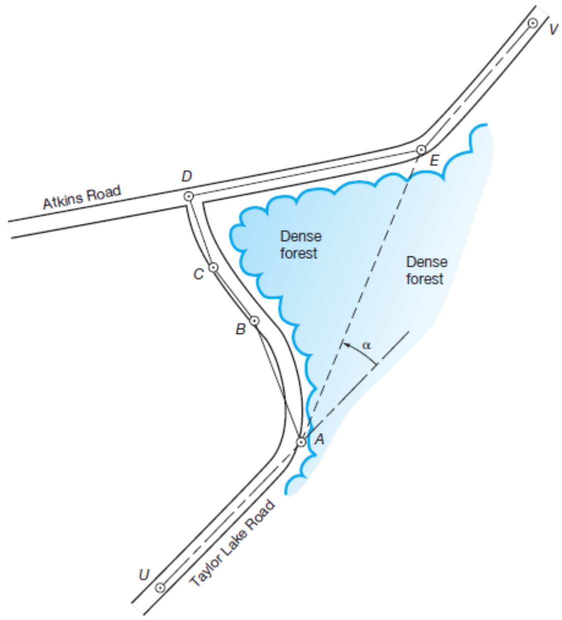
$$Y_B - Y_A = 43{,}755.98 - 43{,}280.21 = 475.77 = \Delta Y$$

$\tan \text{azimuth}_{AB} = 930.93/475.77 = 1.95668075$; $\text{azimuth}_{AB} = 62°55'47''$.

$$\text{length}_{AB} = \sqrt{(930.93)^2 + (475.77)^2} = 1045.46 \text{ ft.}$$

Line	Adjusted		Adjusted	
	ΔX	ΔY	Length (ft)	Azimuth
AB	930.93	475.77	1045.46	62°55′47″
BC	657.94	−762.80	1007.35	139°13′16″
CD	756.57	483.34	897.78	57°25′38″
DE	−313.80	907.99	960.68	340°56′06″

Example:
Compute the length and azimuth of closing line *AE* and deflection angle α of the figure, given the following observed data:



From the coordinates of points A and E, the ΔX and ΔY values of line AE are

$$\Delta X = 7{,}004.05 - 10{,}000.00 = -2{,}995.95\,\text{ft}$$
$$\Delta Y = 17{,}527.05 - 10{,}000.00 = 7{,}527.05\,\text{ft}$$

the length of closing line AE is

$$\text{length}_{AE} = \sqrt{(-2995.95)^2 + (7527.05)^2} = 8101.37\,\text{ft}$$

Point	Azimuth	Departure	Latitude	*X* (ft)	*Y* (ft)
U					
	North (assumed)				
A				10,000.00	10,000.00
	295°18′25″	−2988.53	1413.11		
B				7011.47	11,413.11
	276°42′36″	−1849.64	217.61		
C				5161.83	11,630.72
	301°32′45″	−1627.93	999.39		
D				3533.90	12,630.11
	35°19′22″	3470.15	4896.94		
E				7004.05	17,527.05

$$\tan \text{azimuth}_{AE} = \frac{-2995.95}{7527.05} = -0.39802446;\ \text{azimuth}_{AE} = 338°17'46''$$

$$-\alpha = 338°17'46'' - 360° = -21°42'14''\text{(left)}$$